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TECHNICAL NOTE

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TENSION PROPERTIES OF ALUMINUM ALLOYS IN THE
PRESENCE OF STRESS-RAISERS

I - EFFECTS OF TRIAXIAL STRESS STATES ON THE FRACTURING
CHARACTERISTICS OF 24S-T ALUMINUM ALLOY

By A. W. Dana, E. L. Aul, and G. Sachs

Case Institute of Technology



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SUMMARY

The effects of circumferential notches on the fracturing characteristics of 24S-T aluminum-alloy tensile test bars were investigated. Two variations in notch contour, namely, notch radius and notch depth, were studied. In addition, two different surface conditions were investigated.

Specimens machined after heat treatment were found to be less strong and ductile than those machined before heat treating, if the notch was sharp (small radius). Specimens with comparatively mild notches showed no effect of surface condition. The conclusion was drawn that fracturing began in the center of mildly notched test bars and at the notch bottom of sharply notched test bars.

The average values of fracture stress were transformed analytically into local stress values in the center of the specimens. For bars which fractured in the center, these values become actual fracture stresses, and the corresponding strains, actual ductilities. These fracturing characteristics, for mildly notched bars, were found to depend only upon the transverse tension, which was derived analytically. The fracture stress was found to increase with increasing transverse tension, or increasing triaxiality, whereas the ductility decreased correspondingly.

INTRODUCTION

It is recognized that the behavior of a structure in service cannot be predicted from the metal characteristics obtained by means of simple tests such as tensile or compression tests. On the other hand, the determination of certain quantities by means of notched specimens has been found frequently to yield significant information.

Notched-bar tests, however, present an extremely complex problem. Both the magnitude of the stresses, or stress distribution, and the

ratios between the three principal stresses, or stress state, are not uniform but vary within the metal volume of a part with a contoured surface. The simplest cases of such structures (provided with a notch or stress-raiser) possess rotational symmetry, as represented by cylindrical bars with an external circumferential notch. In view of this, numerous investigations have been conducted on notched tensile test bars. However, up to the present, the analysis of this test has progressed only to a rather preliminary state.

For the elastic region, the stresses have been determined analytically by Neuber (reference 1) and the results of this analysis are generally accepted. A maximum longitudinal stress occurs at the locations of maximum curvature (minimum radius) of the surface. The ratio between this stress peak and the nominal stress derived from the elementary theory of elasticity (disregarding the effects of surface contour) is called stress concentration. The state of stress is uniaxial or biaxial at the surface where the normal stress is zero but becomes triaxial with all three principal stresses being tensions over most of the notched section.

When plasticity occurs, the stress pattern changes rapidly (references 2 to 4). The stress concentration is eliminated by straining of the order of 1 to 2 percent. The original stress distribution is gradually replaced by an entirely different one which can be approximately derived from the laws of plasticity. On the other hand, the state of stress at each point of the notched section apparently changes only slowly with progressive straining. This is evidenced by the fact that the average triaxiality, that is, the ratio of the average transverse to longitudinal stresses, has been found to remain almost constant up to strains of appreciable magnitude. It was further observed in tests on heat-treated steels that the stress concentration on the one hand and the average triaxiality on the other hand depend very differently upon geometrical factors such as the shape and the depth of the notch. These relations permitted over-all analysis of some features of the process of fracturing in both completely plastic and in comparatively brittle steel conditions (reference 5). At present, such data available for notched bars tested in static tension are very incomplete and difficult to analyze, and the results of the analysis are subject to criticism. The progress made to date, however, clearly indicates that this approach to the problem can be progressively expanded to yield more definite and significant results.

Furthermore, the effects of triaxiality and stress-raisers on the fracturing may not follow the same pattern for different metals. From the known features of the stress state in such structures, it appears clear that there exist at least two potential loci of fracturing. At the surface, the stress is high in the elastic region. This condition may be partially retained in a sharply notched bar after plastic flow occurs. Then, appreciable plastic strains develop at the surface while

the stress distribution becomes gradually more uniform. Therefore, the ductility of the metal at the surface, under the conditions of biaxiality present, is exhausted more rapidly than that in the core, and the part should begin to fracture at the surface. On the other hand, fracturing in mildly notched shapes such as represented by the neck of a tensile test bar has been observed to occur in the center (references 6 and 7). This is explained by the fact that with increasing triaxiality the tensile stress required for plastic flow increases, whereas the ductility decreases. Regarding the occurrence of these two types of failure, all available evidence indicates that the likelihood of surface fracturing increases with the sharpness of the notch. It is also clear that truly brittle materials always fracture at the locus of stress concentration, that is, at the surface (reference 8). The occurrence of fracturing in the core requires that most of the stress concentration be eliminated by the plastic flow preceding fracturing to permit the longitudinal stress in the core to exceed that at the surface.

Quantitative information regarding the described relations can be derived only from further extensive experimental investigation. As a contribution to this problem, the investigation on notched tension bars of a heat-treated aluminum alloy was carried out. According to the previous tests on heat-treated steels, the testing of specimens provided with notches of various radii offered the possibility of distinguishing between the effects of stress concentration and the effects of the triaxiality. In order to determine the locus of failure, the metal was investigated with two different surface conditions. These should influence the results if fracturing occurred at the surface but be of no effect if fracturing originated in the interior. In addition, previous tests showed that if the notch depth is varied with a constant notch sharpness, the average triaxiality increases approximately linearly with increasing notch depth up to at least 90-percent notch depth. The investigation of specimens with various depths, therefore, offers a further opportunity to study the effect of triaxiality over a wide range.

For conditions where fracturing occurs in the core, it should then be possible to determine, with a certain degree of accuracy, the stresses at this point and at the moment of fracturing on the basis of previous investigations by Bridgman (reference 9) and Davidenkov (reference 10). The analytically derived data would finally yield the dependence of the fracture stress, that is, of the actual longitudinal stress at the moment of fracturing, upon two variables, the triaxiality and the plastic strain preceding failure.

The aluminum alloy selected for the present investigation appears particularly suitable because it fractures in a regular tensile test with little necking. This permits establishing the basic stress-strain relations in uniaxial tension, or flow stress, for the metal with a high degree of accuracy. In addition, the fracture loads and strains can be measured more accurately for metals which develop no or only a small neck than for metals which neck deeply before fracturing.

This paper comprises an extension of previous investigations on notched-bar tensile tests to aluminum alloys along more fundamental lines. Various members of this laboratory have contributed to the program in general and to the investigation on aluminum alloys in particular. Particular acknowledgement is made to Mr. W. F. Brown, Jr. for his cooperation in establishing the program and in clarifying the results of the investigation and to Mr. M. H. Jones for the help rendered in the experimentation.

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SYMBOLS

R	radius of curvature of notch
a	half the diameter of notched section
s_1, s_2, s_3	principal true stresses (actual)
s_1', s_2', s_3'	principal true stresses (average)
s_f	actual fracture stress for any stress state
s_f'	average fracture stress for any stress state
k	variable flow stress in pure tension
k_f	fracture stress in pure tension
f	variable fracture stress (function of stress and strain state); with subscript zero, in pure tension
e_1, e_2, e_3	principal conventional (unit) strains
$\epsilon_1, \epsilon_2, \epsilon_3$	principal natural strains ($\epsilon = \log_e(1 + e)$)
q	reduction in area or contraction in area at fracture
ϵ_f	maximum natural strain at fracture under conditions of testing; with additional subscript zero, in pure tension

MATERIAL AND PROCEDURE

For the experimental investigation, commercial $\frac{3}{4}$ -inch-diameter 24S-T aluminum-alloy rod was selected. All specimens were re-solution-heat-treated either before or after final machining to insure a high degree of uniformity. This standardizing heat treatment consisted of: (a) Soaking for 1 hour at $920 \pm 10^\circ$ F in an electric Lindberg cyclone forced-convection furnace, (b) quenching in water at room temperature, and (c) room-temperature aging for 4 to 5 days.

Figure 1 illustrates the various specimen types. Cylindrical specimens served to determine the basic properties of the material. Most notched specimens used in the investigation were provided with 60° , V-type circumferential notches. Both the radius at the notch bottom and the notch depth (percent cross-sectional area removed by the notch) were varied within wide limits. For very large notch radii, the notch contour was entirely circular. All specimens possessed the same diameter, 0.212 inch, at the notched section, but the cylindrical diameter was made variable to yield different notch depths.

It was thought that the surface condition would affect the fracture properties of the metal. Machining of the notch results in severe cold working of the metal at the notch bottom. Consequently, a reduction in ductility should be expected if fracturing occurs at the surface. In order to confirm or discount this conception, specimens provided with notches having a 50-percent notch depth were used. The radii employed ranged from sharp (notch radius less than 0.0005 inch) to 2.000 inches. The variation in surface condition was obtained by one of the two following preparation schedules:

Schedule I

- (a) Re-solution-heat treatment of the as-received rod
- (b) Machining of specimen, including the notch

Schedule II

- (a) Machining of specimen, including the notch
- (b) Re-solution-heat treatment

Schedule I was designed to produce a layer of severely cold-worked material at the notch surface, whereas schedule II eliminated this effect. As a result of this study, schedule II was used for the remainder of the investigation.

The specimens were tested in specially designed fixtures that yield an eccentricity of less than 0.001 inch. These fixtures and the test procedure used have been previously described (reference 3). The speed of testing was sufficiently slow to enable recording of stress-strain curves. This speed corresponded to a rate of travel of the testing-machine head of about 0.02 inch per minute. The total testing time varied between 5 and 15 minutes.

The change in diameter of the notched section, or average transverse strain, was obtained by means of a radial strain gage improved over that previously described (reference 3). The new gage (see fig. 2) permitted measurements of changes in diameter with an accuracy of ± 0.0001 inch. The principal changes in the gage were: A larger dial indicator, springs that act in the same plane as the knife edges, stops to prevent the knife edges from coming together upon fracture of the specimens, and aluminum holders for the knife edges to reduce the weight. The average contraction in area at the root of the notch, or "notch ductility," could be determined from such strain measurements with an estimated accuracy of ± 0.1 percent.

The results of the tests on the 24S-T specimens heat-treated after machining are assembled in table I and on those machined after heat treating, in table II. The quantities given in these tables are the "notch strength," that is, the maximum load recorded divided by the original area of the notched section; the "notch ductility," that is, the percent reduction in cross section at the root of the notch at the fracturing point; and the "fracture stress," that is, the ratio of the breaking load to the area of the notched section at failure. The term "notch sharpness" a/R is defined as the ratio of half the notch diameter a to the radius of curvature R at the root of the notch.

The notch strength characteristics of 50-percent-notched specimens are represented graphically as functions of the notch sharpness¹ in figure 3 for both surface conditions investigated and in figure 4 for various notch depths. In addition, figures 5 to 8 show the stress-strain curves for a selected number of specimens.

In conjunction with the investigation of the effect of surface condition, a few specimens that were re-solution-heat-treated after machining were investigated to determine the loci of fracturing, that is, the core or the outer fiber of the tensile test bar. Specimens

¹In this representation and in those following, in which notch sharpness is one of the variables plotted, the scale is based on the quantity $\frac{a/R}{\frac{a}{R} + 10.0}$. The value 10.0 is the a/R value chosen to fall in the middle of the plot. This scale spreads the experimental points and allows for the plotting of $\frac{a}{R} = \infty$.

provided with various notch radii (sharp, 0.007-, 0.029-, 0.060-, and 0.125-inch) and having a 50-percent notch depth were subjected to tension until failure was imminent. The tests were then stopped and the specimens sectioned longitudinally, mounted in Bakelite, polished down to the center line, and etched.

RESULTS AND DISCUSSION

Crack Formation

Figures 9 to 12 illustrate longitudinal sections through several notched test bars strained very close to failure. In any of the sharply notched specimens (figs. 9 to 11) a crack is visible at the notch bottom. This crack is rather deep and wide for the sharpest notch (fig. 9); this indicates that it started at a load considerably lower than the maximum load observed in the test. Furthermore, the photographs for the sharply notched bars represent the conditions at maximum load. The fact that it was possible to interrupt the test at this point leads to the conclusion that crack propagation in such shapes is a rather slow process. This is also confirmed by the observation that complete separation of such test bars occurred rather smoothly, that is, with little noise.

With decreasing notch sharpness, the crack apparently starts later and is less wide (figs. 9 to 11). Then, below a certain notch sharpness, all attempts to detect a crack failed. For the investigated 24S-T specimens, re-solution-heat-treated after machining, either surface cracks or internal cracks² were absent if the notch sharpness was 1.8 ($R = 0.060$) or smaller. In these tests, it was not possible to reach the maximum load without subsequent failure on unloading. Therefore, the tests were interrupted at a load possibly 5 to 10 percent below the expected maximum. Also, fracturing of such specimens occurred with an appreciable noise. These observations indicate that crack propagation in mildly notched bars was a very rapid process in the alloy investigated. It appears significant that, in necked tensile test bars of more ductile alloys, internal cracks have been found repeatedly. It appears reasonable to associate the sudden failure of a mildly notched 24S-T test bar with fracturing in the core. This is the case for the comparatively brittle alloy investigated.

The foregoing conclusions are further supported by the fact that the ratio between the stresses required to produce a certain strain in a particular notched bar and in the cylindrical specimen (fig. 13) was

²One specimen showed an indication of a beginning internal crack, the presence of which, however, could not be definitely established.

generally almost constant for strains above a certain value. However, sharply notched bars (fig. 14) exhibited a deviation to lower stress ratios approaching failure, which started earlier and was more pronounced the sharper the notch.

Effects of Notch Sharpness

The distributions of both the longitudinal tension (stress concentration) and of the stress state are in the elastic state determined primarily by the notch sharpness (reference 1). The test data obtained for notched-bar tensile tests on ductile metals (references 2 to 4) indicate that these effects of notch sharpness also apply qualitatively after plastic flow occurred.

The notch sharpness for a given notch depth affects very considerably the strength characteristics of tensile test specimens. In the case of the investigated 24S-T alloy, these notch strength characteristics follow approximately the trend previously observed on other ductile alloys. With increasing notch sharpness, the notch ductility decreases continuously. Both the notch strength and the average fracture stress, however, exhibit a maximum at a certain notch sharpness. This maximum is more pronounced and occurs at a smaller notch sharpness for the fracture stress than for the notch strength.

It appears that the notch sharpness at which the fracture stress is at a maximum roughly subdivides the entire range of notch sharpnesses into ranges of mild and sharp notches. The evidence available at present indicates that similar relations apparently apply to all metals in the range of mild notches. On the contrary, the notch strength characteristics may assume very different trends for different metals in the range of sharp notches. These relations have been made the subject of an extensive investigation. In this first paper, an attempt is made to establish more definitely different ranges of notch sharpness and to analyze the effects of mild notches.

Effects of Surface Condition

Figure 3 permits a comparison of the notch strength characteristics of test bars which differ only regarding their surface condition. These data clearly show that the two series of test data differ more with increasing notch sharpness. However, below a certain notch sharpness, the strength characteristics were found to be practically independent of the surface condition.

For the notch shapes where the surface condition influences the test data, a reduction in all strength characteristics and ductility by machining after heat treating is observed. These effects are readily explained by the fact that under such conditions fracturing occurs at

the surface. Then, the ductility of the surface at the notch bottom determines both the average ductility and the fracture stress. Because of the small ductility values observed for the alloy investigated, the notch strength also exhibits the same differences as the fracture stress.³

Regarding the stresses and strains at the notch bottom of a sharply notched specimen subjected to plastic flow, the following facts are established. The longitudinal strains at the notch bottom are larger, the sharper the notch, for a given average strain, other conditions being equal. The stress state at the notch bottom is probably close to that of plane strain, that is, no strain in the circumferential direction, because of the restraint by the less plastic core. The ductility of the surface fiber should be then constant for a given surface condition, that is, the same for all specimens with relatively sharp notches. The notch ductility and the average fracture stress then become primarily functions of the initial stress concentration, which also determines (in an as yet unknown manner) the stress and strain concentrations retained at the moment the surface fiber fractures.

The experimental data for the sharply notched bars, however, have little basic significance. Because of the slow crack propagation, discussed in the preceding section, actual fracturing, that is, crack formation, occurs at some lower stress and strain values than the measured ones. The latter apply to the termination rather than to the beginning of the cracking process. The data available at present are not sufficient to analyze the fracturing phenomena in sharply notched bars. Therefore, such data are not further discussed in this paper.

Relations between Notch Strength Characteristics for Mildly Notched Bars

As a result of the previous discussions of sharp notches, the following section is devoted primarily to mild notches where fracturing may be assumed to occur in the center. This range of notch sharpnesses is limited on one end by the regular tensile test. It has been definitely established that fracturing in a ductile tensile test specimen which necks before failure starts in the center. This neck constitutes the mildest notch (largest notch radius) which can be realized experimentally. In the case of the investigated alloy, the neck is rather shallow because of its limited ductility. Therefore, experimental data can be obtained for very mild notches.

The upper limit of the range of notch sharpness, where a notch can be considered as mild, is not known at the outset. For the purpose of this analysis, a mild notch may be defined as a notch where the effect of any

³Notch strength (ultimate strength) and fracture stress for tensile tests are only correlated if fracturing occurs without necking.

stress concentration at the notch bottom becomes insignificant. This definition excludes immediately all notch shapes where fracturing occurs at the notch bottom, that is, according to figures 9 to 12, all notch sharpnesses of greater than approximately 1.8 ($R < 0.060$ inch) for 50-percent notch depth. As expected, the surface condition was also found to be without effect for notches having a sharpness below this limit.

The absence of these phenomena does not mean necessarily that the stress concentration at the notch bottom is nil and without effect on the notch strength characteristics. However, very ductile metals exhibit a deep, sharp-radius neck without any indication of stress-concentration effects. Consequently, a less ductile metal should behave similarly and, particularly, fracture in the center if specimens are tested, the notch sharpnesses of which do not exceed that possible in regular tensile tests.

The stress pattern in a necked tensile test bar has been analytically determined and is discussed in the following section. If the results of this analysis should be applicable to notched tensile test bars, they would require that the various notch strength characteristics be correlated. Or, in other words, if the notch strength and fracture stress are plotted against the notch ductility, as shown in figures 3 and 4, definite relations should exist between these quantities in the range of mild notches. According to figure 4, this is found to be true within a certain range of notch sharpness and notch depth. Up to a certain notch sharpness, which is smaller the smaller the notch depth, both the notch strength and the fracture stress are functions of the ductility only, irrespective of the shape of the notch. Either a comparatively deep but mild notch or a sharper but shallow notch may yield the same values of all three notch strength characteristics. This results from the fact that if fracturing occurs in the interior and the disturbing effect of any stress concentration is insignificant, both the notch ductility and the average fracture stress are determined by the actual values of ductility and fracture stress at the center. If both the stress distribution and one stress value were known, this actual fracture stress could be computed. The quantities required for such an analysis are available either by experimentation or by calculation.

Stress-Strain Relations of Notched Specimens

Tests on cylindrical specimens yield the stress-strain curve in regular tension or for zero notch sharpness. From this stress-strain curve the flow stresses k can be obtained by means of Bridgman's analysis. (See section entitled "Flow Stress Curve.") These flow stresses deviate from the regular tensile stresses to lower values after a neck has been formed, as illustrated in figure 5.

Figure 5 also shows stress-strain curves (curves of average tension against reduction in area) for a number of mildly notched bars. Because of the presence of transverse tensile stresses in the notched section, the tension required to produce a given strain increases with certain factors, such as the notch sharpness and notch depth. However, this only applies to strains exceeding a small amount, say, 1 percent or 2 percent, which is necessary to eliminate the initial stress concentration. The dependence of the average tension upon the notch sharpness, for 50-percent-notched test bars, is further illustrated in figures 6 to 8 for the entire range of notch sharpnesses investigated.

The ratio between the average applied stress s_1 and the flow stress k may be introduced as "average triaxiality." The change of this average triaxiality with progressing straining is shown in figure 13 for the mildly notched bars, the stress-strain curves of which are given in figures 5 and 7. It is interesting to note that the average triaxiality in general decreases slowly with increasing strain.⁴

For the few specimens, the ductility of which exceeds approximately 18 percent, the average triaxiality again increases at still higher strains (fig. 13). This is obviously caused by a process of necking which begins in notched specimens at practically the same strains as in unnotched specimens. This necking would cause a gradual increase in notch sharpness, whereas at smaller strains the notch contour probably changes only slightly. For such specimens which exhibited this increase in triaxiality at high strains, the applied tensile load also passed through a maximum value at the necking strain. This is in conformance with the theory of necking which considers this phenomenon as a problem of instability (reference 11).

Table III gives the various values of necking strains for the notched bars with different notch radii and notch depths. The necking strains were calculated by means of a method based on the consideration of necking as an instability phenomenon (references 11 and 12). Figure 15 illustrates the results obtained by this method for a number of specimens which possessed sufficient ductility to be subject to necking.

ANALYSIS OF FRACTURING OF MILDLY NOTCHED TENSILE TEST BARS

The stresses and strains occurring in elastically loaded structures of various shapes can be determined with appreciable accuracy (reference 1). On the other hand, little definite knowledge is available regarding the stresses and strains occurring in such shapes after their elastic limit is exceeded; then the result is first a semiplastic and then a completely plastic condition of their minimum cross section.

⁴The difference between the curves for the two surface conditions does not exceed the scattering expected from differences in the metal condition.

In the following analysis only the completely plastic region is considered. A condition which must then be fulfilled, according to previous investigations (reference 5), is that the average plastic strains for such a cross section must exceed a certain minimum value, say, 1 or 2 percent.

In order to analyze completely the fracturing phenomena occurring in ductile, notched-bar tensile tests, the actual stresses and strains must be known. However, at present no data of this type are available for sharply notched test bars. On the other hand, for rather mild notches, such as those represented by the shapes of initially cylindrical tensile test bars subjected to large strains, the distributions of stresses and strains have been investigated analytically and experimentally. The results of these investigations can be applied to advance the knowledge of the notched-bar test beyond its present rather unsatisfactory state.

In order to achieve the aforementioned purpose, the regular tensile test is first analyzed in a more detailed manner than has been done up to the present. Previous conceptions advanced and widely accepted regarding the fracturing in this test appear incorrect. These conceptions are revised to serve as a basis for the analysis of the notched-bar tensile test. For this analysis, the stresses and strains are always related to the cross section upon which they act. However, the so-called true stresses and true strains thus obtained as test results generally represent average values. These average values must be transformed into local or actual values by some process of analysis or deduction to attain physical significance. The following discussions are devoted to this problem. Primarily, such conditions of geometrical shape and metal are considered which should result in fracturing at the center of a test bar.

Condition of Fracturing

The major objective of an analysis of fracturing of mildly notched tensile test bars is the establishment of a condition of fracture. Such a condition in its most general form should relate a stress function, preferably the largest principal stress at the moment of fracturing or fracture stress ($s_1 = s_f$) to pertinent fundamental variables. Some variables such as the metal, the temperature, and the rate of straining (speed) can be readily kept constant in a particular series of tests. Then, two major variables are retained and cannot be separated by any simple procedure, namely, (1) the state of stress and (2) the magnitude of the plastic strain preceding fracturing.

The state of stress can be defined by the ratios between the three principal stresses s_2/s_1 and s_3/s_1 or, in the case of rotational symmetry treated herein, that is, $s_2 = s_3$, by the ratio between the the two extreme principal stresses s_3/s_1 .

In order to facilitate calculations, the stress state in the following deductions is measured by the derived quantity s_1/k . In this quantity, the flow stress k is defined by the condition of plasticity (for rotational symmetry)

$$s_1 - s_3 = k \quad (1)$$

The flow stress k is consequently equal to the applied tensile stress (yield strength) in pure tension, where $s_3 = 0$. The two major variables measuring triaxiality are thus related by the equations

$$\frac{s_1}{k} = \frac{s_1}{s_1 - s_3} = \frac{1}{1 - \frac{s_3}{s_1}} \quad (2a)$$

or

$$\frac{s_3}{s_1} = 1 - \frac{k}{s_1} \quad (2b)$$

The magnitude of plastic strain can be measured by various functions of the three principal strains e_1, e_2, e_3 . In the case of rotational symmetry

$$e_2 = e_3 = \sqrt{\frac{1}{1 + e_1}} - 1 \quad (3a)$$

or, in terms of natural strains,

$$\epsilon_2 = \epsilon_3 = -\frac{\epsilon_1}{2} \quad (3b)$$

where

$$\epsilon = \log_e(1 + e) \quad (4)$$

Usually the strain in tensile tests is measured by e_1 , ϵ_1 , or the so-called reduction in area q defined by the following equation:

$$1 - q = \frac{1}{1 + e_1} \quad (5a)$$

or

$$\log_e(1 - q) = -\epsilon_1 \quad (5b)$$

The fracture characteristics depend upon further variables, the effects of which are usually neglected. These variables are variations in the stress and strain states. The strain state can be defined by the ratios

$$\frac{d\epsilon_2}{d\epsilon_1} \quad \text{and} \quad \frac{d\epsilon_3}{d\epsilon_1} \quad (6)$$

In the case of rotational symmetry, equation (3b) yields

$$\frac{d\epsilon_2}{d\epsilon_1} = \frac{d\epsilon_3}{d\epsilon_1} = -\frac{1}{2} \quad (7)$$

Then, the strain state is always the same. In notched tensile test bars the center fiber is generally in a state of rotational symmetry. Its strain state is therefore always constant, that is, independent of both the shape of the part and the magnitude of strain.

However, for a given strain state, the stress state is not definitely determined but may be widely different. The stress state in a cross section, which is in a condition of plastic flow, depends primarily upon the surface contour in its vicinity. Therefore, the stress state not only is different for different shapes of the test specimens, but it also changes with progressing straining, the amount of change depending upon the magnitude of the contour change. The contour change and consequently also the change in stress state are particularly large for very ductile, regular tensile test bars which develop a deep neck. Under such conditions, it must be considered that this change from uniaxial tension to a high degree of triaxial tension also influences the fracturing conditions. According to Bridgman's tests on the effects of superimposed pressure (reference 13), it must be assumed that the fracture stress and strain measured at a given triaxiality are distorted

to increased values if the previous history consists of stress states of lower triaxiality. However, in the case of the comparatively brittle metals investigated herein, this effect of stress history should be negligible.

Flow Stress Curve

The flow stress k is obtained by means of a regular tensile test. The stress-strain curve in regular tension (fig. 5) yields the flow stress directly up to the moment of necking. In the case of the aluminum alloy investigated, the necking strain was observed to be approximately 18 per cent. Up to this strain, the flow stress k is equal to the average tensile stress s_1' , which in turn is equal to the actual tensile stress s_1 at any point of the cylindrical section,

$$k = s_1' = s_1 \quad (8)$$

However, after necking occurs, these three quantities become distinctly different from each other. Also, the actual tensile stress varies over the cross section. In the following discussions, only the center fiber is considered, s_1 being the longitudinal stress for this fiber.

Various attempts (references 9, 10, and 14) have been made to calculate the flow stress and the actual stress from the average stress and to correlate these stress values with the neck contour and magnitude of plastic strain. For the following analysis, Bridgman's equations (reference 9) have been found suitable:

$$k = s_1' \left[\frac{1}{\left(1 + 2\frac{R}{a}\right) \log_e \left(1 + \frac{1}{2} \frac{a}{R}\right)} \right] \quad (9)$$

$$s_1 = s_1' \left[\frac{1 + \log_e \left(1 + \frac{1}{2} \frac{a}{R}\right)}{\left(1 + 2\frac{R}{a}\right) \log_e \left(1 + \frac{1}{2} \frac{a}{R}\right)} \right] \quad (10)$$

where a is the half diameter of the necked section and R is the contour radius of its surface.

Because of the small amount of necking encountered for the aluminum alloy, the radius R was measured after fracturing only; this yielded

the flow stress for a strain equal to the fracture strain (fig. 5). The flow stress curve is then determined over the entire range of strains with sufficient accuracy for the present purpose.

Formal Analysis of Fracturing

The phenomenon of fracturing is, according to Ludwik (reference 7), formally explained as the intersection of two functions, the function of the stress required for plastic flow against strain and the function of the fracture stress against strain (fig. 16). If a metal is subjected to increasing load, it will flow if, for zero plastic strain, the fracture stress exceeds the stress required for plastic flow. It must be then assumed that the fracture stress increases with increasing strain slower than the stress required for plastic flow. This results in an intersection of the two functions at a given stress, the fracture stress, and, at a given strain, the ductility. At this point of fracturing, therefore, the fracture stress and stress required for plastic flow are identical.

It is common practice to define the actual fracture stress s_f as the largest principal stress s_1 present at the moment of fracture. The fracturing phenomena which are of primary interest occur if the fracture stress is a tension. Only this region of fracturing has been considered by previous investigators to any extent.

The condition of fracturing in its most common form correlates the actual fracture stress s_f to its value $s_{f0} = k_f$ in uniaxial tension. This relation, however, neglects the differences in fracturing strain. A more basic condition of fracture would, therefore, be one which correlates the hypothetical fracture stress f for a given strain to its values f_0 under conditions of uniaxial tension.

Fracture Stress for Uniaxial Tension

For the subsequent analysis of notched test bars, approximate values of the fracture stress function f_0 under conditions of uniaxial tension are desired.

Regarding this function, fracture stress against strain, it has been shown first by Davidenkov and Sakharov (reference 15), by means of brittle fracturing at low temperatures, that the fracture stress increases with increasing strain. The exact nature of this function is unknown as yet. For most purposes, it will be satisfactory to assume that the rate of increase in fracture stress is slightly less than that of the stress required for plastic flow.

A suitable function conforming to the experimental evidence would be the following:

$$f_0 = \frac{k + k_f}{2} \quad (11)$$

where $k_f = s_{f0}$ is the fracture stress in uniaxial tension. However, at the present time, this basic constant is not known. It appears, therefore, permissible, for metals of rather limited ductility, to substitute for k_f the average fracture stress s_f' , yielding

$$f_0 = \frac{k + s_f'}{2} \quad (12)$$

This inaccuracy is of little significance in comparison with the magnitude of the effects considered.

Fracturing Characteristics for Regular Tension

It has been repeatedly demonstrated that fracturing in regular tension, after the development of a neck, occurs in the center fiber. The fracture stress s_f and fracture strain or ductility ϵ_f of this fiber, therefore, comprise fracturing characteristics for a certain condition of triaxiality.

The strain state at various points of the necked section of a tensile test specimen, according to Davidenkov (reference 10), is practically uniform. The average fracture strain in regular tension is, therefore, equal to the fracture strain. This ductility can be measured by the common contraction in area q_0 or by the natural strain $\epsilon_{f0} = -\log_e(1 - q_0)$. (See equation (5b).)

The fracture stress is now determined by equations (9) and (10) which can be combined to yield, for $s_1 = s_f$,

$$s_f = k \left[1 + \log_e \left(1 + \frac{1}{2} \frac{a}{R} \right) \right] \quad (13)$$

The flow stress k in this equation is that for $\epsilon = \epsilon_{f0}$ (see fig. 16).

The triaxiality present in regular tension at the moment of fracturing is then determined by the value s_f/k .

Fracture Stress Function for Regular Tension

The process of fracturing in a regular tensile test can now be explained formally as follows. (See fig. 16.)

The stress s_1 required for plastic flow of the center fiber, which is the locus of fracturing in a regular tensile test, is known over the entire range of strains. Up to the moment of necking, this stress is equal to the flow stress, that is, $s_1 = k$. For larger strains it is given by equation (13), because it is a function of both the strain and the neck contour. Because of the progressive development of the neck, this stress s_1 increases with increasing strain at a considerably higher rate than the flow stress.

The fracture stress f should be located somewhere between the stress required for plastic flow s_1 and the actual fracture stress for the same state of stress. Up to the point of necking, values halfway between the average stress s_1' and the average fracture stress s_f' will be rather close to these unknown values of fracture stress in uniaxial tension ($f = f_0$). By the same reasoning, the fracture stress for larger strains can then be taken as the average of the actual fracture stress s_f and the actual stress s_1 .

The resultant functions of fracture stress f and the stress required for plastic flow s_1 against strain ϵ_1 are illustrated for the investigated aluminum alloy in figure 16. The reversal in curvature in both curves, beyond the point of necking, results from the progressive increase in triaxiality. Regarding the fracture stress, this trend means that the fracture stress should increase with both increasing strain and increasing triaxiality.

It is rather common practice to represent the two functions f and s_1 for a tensile test in an entirely different manner. (See fig. 17.) Following Kuntze's suggestion (reference 16), it has been assumed that the fracture stress increases sufficiently fast before necking to exceed at this point the average fracture stress. Then, in order to obtain coincidence of the fracture stress and average stress for plastic flow at fracturing, the fracture stress has been assumed to decrease with further straining. This phenomenon has been explained physically as a progressive deterioration of the metal by the process of necking. However, the previous discussion clearly shows that beyond the necking

point the representation of average stresses has no physical significance. The analysis of notched-bar tensile tests will also reveal that no deterioration occurs in contours corresponding to those of a neck. On the contrary, the fracture stress will be found to be temporarily increased because of the presence of triaxiality. Furthermore, the conception that the fracture stress initially increases rapidly to rather high values has been derived from an incorrect evaluation of notched-bar test results and does not conform to general knowledge.

Regarding the fracture stress and stress under conditions of truly uniaxial tension, no definite conclusions can be drawn from a conventional tensile test. According to some tests by Kuntze (reference 17), where the neck was repeatedly eliminated by machining to a cylindrical shape, the fracture strain becomes considerably larger than that derived from the conventional reduction in area in the neck at fracturing (contraction in area). For large contractions in area, the strain must be extremely high to result in fracture stresses anywhere near the actual (or even the average) fracture stress observed, as illustrated in figure 5. Thus, it appears that many metals would possess an almost unlimited ductility if subjected to uniaxial tension. Fracturing of such metals only occurs because of the development of a high degree of triaxiality in the neck, which progressively reduces the ductility until it conforms approximately to that belonging to the particular stress state. This phenomenon is again best explained by assuming that the fracture stress s_f increases with increasing triaxiality, as shown in figure 16.

Method of Analysis of Notched Tensile Test Bars

It is rather doubtful whether Bridgman's analysis of the necking process (reference 9) is entirely correct. In particular, it is known that for a given ratio of notch radius to diameter of the notched section the magnitude of the stresses depends considerably upon the depth of the notch (reference 5). However, the results of the analysis may be considered to be rather accurate regarding the stress distribution.

It can be then assumed that the same type of stress distribution also occurs in any mildly notched section. Furthermore, for such shapes, it may also be assumed, according to Davidenkov's measurements,⁵ that the strains are practically uniformly distributed. They are, then, for any point of the notched section, equal to the average strain, or notch ductility. The notch ductility ϵ_f can be readily measured in notched-bar tensile tests.

Regarding the stresses, a notched-bar test yields the average fracture stress $s_1' = s_f'$. In order to obtain the actual fracture

⁵Investigated notch sharpnesses up to approximately 1.0.

stress $s_1 = s_f$ determined by equations (9) and (10), the flow stress k for the fracture strain $\epsilon = \epsilon_f$ must also be known. This stress can be taken from figure 5. Then, by means of figure 18, both the actual fracture stress $s_1 = s_f$ and the triaxiality at the moment of fracturing s_1/k are determined.

The foregoing process of analysis has been applied indiscriminately to the results of all notched-bar tensile tests conducted on 24S-T aluminum alloy. If these results apply to a process of fracturing at the center fiber, both the fracture stress and the ductility should be definite functions of the triaxiality. However, if fracturing occurs at the surface, as is to be expected for sharply notched test bars, the center fiber would not have reached the condition of fracturing at this moment. Consequently, the analysis must yield for such conditions both stress and strain values below those for mild notches, where fracturing in the center fiber is probable. Furthermore, for sharp notches the surface strains are high and the average strain is, therefore, larger than the strain of the center fiber. However, this difference is probably insignificant in comparison with that between the strain at the moment of fracturing (at the surface) and the ductility of the center fiber.

Measured true stress-strain curves for a number of notched test bars, differing in notch sharpness, notch depth, and surface condition, are again represented in figure 19. Vertically above the termination point of each stress-strain curve is shown a point of fracturing, the abscissa of which is the notch ductility and the ordinate of which is the stress value at the center of the test bar at the moment of fracturing. The flow stress curve in pure tension is also added to this figure.

The physical significance of the points of fracturing is revealed if a large number of such test results are assembled. Thus, figure 20 illustrates the data obtained by this procedure for all 24S-T test bars heat-treated after machining. The results of a few tests on mildly notched test bars machined after heat treating are also included.

Dependence of Fracture Stress and Ductility upon Triaxiality

According to figure 20, the data obtained on mildly notched test bars, including the conventional tensile test, clearly outline a boundary curve. Along this boundary curve the ductility is larger, the lower the fracture stress. For a given fracture stress, the boundary curve yields the highest ductility value obtainable in tests on notched tensile test bars.

A particular point on the boundary curve does not correspond to a particular notch shape but to a variety of comparatively mild notches

which yield the same ductility and, consequently, also the same fracture stress. It is believed, therefore, that such values represent the actual fracture characteristics of the investigated alloy, determined only by the degree of triaxiality present in the center of the test bar at the moment of fracturing. This triaxiality s_f/k can be obtained for each point from the representation in figure 20 by forming the ratio between the ordinate of the point of fracturing and that on the flow stress curve for the same strain.

In figure 21, the fracture stresses and ductilities which define the boundary curve for mild notches are replotted as functions of the triaxiality s_f/k . The ductility in this graph is measured by the maximum natural strain ϵ_f . Regarding the ductility, figure 21 illustrates the well-recognized fact that it decreases with increasing triaxiality. It is, however, interesting to note that, for the metal investigated, an appreciable ductility is retained at the highest degree of triaxiality which may occur in mildly notched sections.

The actual fracture stress increases, according to figure 21, considerably with increasing triaxiality. This increase is only slightly smaller than that of the stress required for plastic flow determined by equation (1):

$$s_1 = k + s_3$$

Curves for this stress at various selected strains (for various values of k) are also added to figure 21. Thus, within the range of triaxialities and ductilities investigated, the fracture stress follows approximately the condition of constant shear stress (equation (1)) which determines the stress required for plastic flow. This confirms the conclusions drawn from previous tests on heat-treated steels on the basis of a less rigid analysis (references 4 and 5).

The actual fracture stresses relate to variable strain values, namely, the fracture strains. In order to obtain a probable relation between fracture stress and triaxiality, for a constant strain, information must be available regarding the effect of strain on the fracture stress. This relation cannot be determined by direct experimentation. However, estimated values of the fracture stress for uniaxial tension can be obtained by a process of reasoning, such as defined by equation (12). In figure 22 the actual fracture stress is again shown as a function of triaxiality. This graph also includes the stresses required for plastic flow to a selected strain value ($\epsilon_f = 0.246$). Furthermore, the estimated fracture stress f_0 in uniaxial tension for the same strain value is added. Then, for this given strain, two values of fracture stress are available, the actual fracture stress for a triaxiality which yields

the selected strain as ductility and the estimated fracture stress for uniaxial tension. These two values determine a probable relation between fracture stress and triaxiality for the constant strain. This relation does not appear very sensitive regarding errors in estimating the fracture stress in uniaxial tension. The position of its trend curve is confined to the rather narrow area between the trend curves of the actual fracture stress and of the stress required for plastic flow. The fracture stress, for any constant strain, therefore, is found to depend upon triaxiality only to a slightly smaller extent than the stress required for plastic flow.

However, such a condition of fracture is not in agreement with the fundamental conceptions on the metallic state. This condition would yield an infinitely large fracture stress if the principal stresses become equal, that is, $s_1 = s_3$. On the contrary, according to any physical or mechanical theory of fracturing, it would be expected that fracturing under such conditions of complete triaxiality would occur at a definite fracture stress. Also, in this stress state all materials should be completely brittle.

In order to check whether the test results are compatible with these conceptions, they are replotted in figure 23, with the quantity $\frac{s_3}{s_1} = \frac{s_3}{s_f}$ as the measure of triaxiality and the ductility expressed as natural strain $\epsilon_f = -\log_e(1 - q)$. In this representation, the limiting triaxiality is represented by an abscissa value $\frac{s_3}{s_1} = 1$. The test results in figure 23 do not permit any decision whether the fracture stress would extrapolate to a finite value for $\frac{s_3}{s_1} = 1$. An alternative law which would yield such a finite value is that the experimentally established relation only applies to ductile metals. Then, the relation may change discontinuously if the metal becomes brittle under high triaxialities.

Effects of Sharp Notches

Previous attempts to analyze notched-bar tensile tests were devoted primarily to sharply notched test bars. As can be seen from figure 20, the fracturing data obtained from such test bars provided with severe stress-raisers cover a wide range of stresses and strains. In figure 20, all such possible combinations of fracturing stresses and strains are represented by a rather definite area. This area is bounded in the region of large values of stresses and strains by the boundary curve for mild notches. This curve represents values dependent only upon the triaxiality in the center of the test bar.

Toward small values of stresses and strains, the boundary condition is apparently determined by data obtained with the sharpest possible notches. Such notches are V-type notches of small flank angles, 60° or less, and various depths. Apparently, the depth is the major variable upon which the position of a particular point on this boundary depends.

For any particular set of test conditions, that is, the effects of a single variable, the test results follow a certain curve within the area between the two boundaries.

The location of the second boundary for a particular alloy appears of appreciable fundamental and commercial importance. It represents the effects of the most severe stress-raisers possible. In addition, the effects of stress raisers of intermediate severity are obviously determined by those of the most severe stress-raisers. Also, under such conditions factors such as the nature of the surface become significant.

CONCLUSIONS

The investigation of tensile test bars provided with circumferential notches of various contours yields results of fundamentally different significance for mildly and for sharply notched specimens. An attempt has been made to analyze the fracturing phenomena in mildly notched bars, by using the procedure developed for regular tensile tests in which necking occurs before fracture. This analysis of mildly notched bars is based on the validity of the following relations:

- (a) Fracture begins at the center of the bar.
- (b) The strain distribution across the notched section is uniform.
- (c) Bridgman's stress distribution is a close approximation to the one present at the notched cross section.
- (d) Fracture stress increases with increasing strain.

The analysis then yields, for each specimen, the values of fracture stress, fracture strain, and triaxiality. For a range of triaxialities which comprises almost the whole range obtainable by means of notched-bar tensile tests, such tests give the following relations between the foregoing three fundamental quantities:

1. The fracture strain decreases continuously with increasing triaxiality. The rate of this decrease probably differs considerably for different materials. The investigated 24S-T aluminum alloy retains an appreciable ductility at the highest triaxialities obtainable in this study.

2. The actual fracture stress increases with increasing triaxiality. For a comparatively ductile metal, such as that investigated, the rate of increase is a large fraction of that of the stress required for plastic flow. No conclusions can be drawn from notched-bar tensile tests regarding the condition of fracture for metals which are brittle within the accessible range of triaxiality.

3. The actual fracture stresses, for various triaxialities, are associated with various strains. For any constant strain, the fracture stress must be located somewhere between the actual fracture stress and the stress required for plastic flow. Therefore, the hypothetical fracture stress for any given strain depends upon triaxiality in almost the same manner as the stress required for plastic flow.

4. The stress required for plastic flow is determined by the condition of plasticity. Consequently, the condition of fracture for ductile metals deviates only slightly from the condition of plasticity. Under conditions of rotational symmetry, the condition of plasticity is identically described by a constant maximum shear stress and a constant distortion energy.

5. In order to account for the decrease in ductility with increasing triaxiality, the unknown condition of fracture must deviate from the maximum shear stress condition to yield lower stresses. This requirement is fulfilled if the condition of fracture is located somewhere between the condition of maximum shear stress and the condition of maximum principal stress (maximum stress condition). For ductile metals, the condition of fracture has been found to be considerably closer to that yielding a constant maximum shear stress than to that determined by a constant maximum principal stress.

Case Institute of Technology
Cleveland, Ohio, June 11, 1947

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TABLE I

RESULTS OF NOTCHED-BAR TENSILE TESTS ON 24S-T ROD

$\left[\frac{3}{4} \right]$ -inch rod re-solution-heat-treated after
machining; $a = 0.106$ inch

Notch diameter (in.)	Notch depth (percent)	Notch radius (in.)	Notch sharpness, a/R	Notch strength (psi)	Fracture stress (psi)	Notch ductility (percent)
0.209	0	∞	0	71.0×10^3	97.0×10^3	32.6
.212	0	∞	0	70.2	100.0	34.2
.225	0	∞	0	69.0	97.5	31.5
.225	0	∞	0	68.8	98.4	34.2
.221	3.0	$<.0005$	∞	71.2	89.9	21.4
.221	3.0	$<.0005$	∞	70.3	85.3	17.9
.229	6.0	$<.0005$	∞	65.5	75.3	13.4
.229	7.0	$<.0005$	∞	64.5	69.3	6.9
.223	12.0	.010	10.6	75.8	89.8	15.0
.222	12.5	.010	10.6	77.0	85.0	12.4
.223	10.0	.023	4.6	76.0	93.0	18.9
.222	12.0	.023	4.6	76.5	91.9	16.8
.221	13.0	.060	1.8	77.1	93.6	17.9
.221	12.0	.060	1.8	76.3	92.4	17.6
.222	12.0	.125	.8	76.6	102.4	26.0
.222	11.0	.125	.8	76.0	103.5	28.3
.222	22.0	$<.0005$	∞	68.8	73.0	5.7
.222	22.0	$<.0005$	∞	68.0	74.9	9.0
.222	21.5	.010	10.6	79.4	88.0	9.8
.222	22.0	.010	10.6	79.9	89.5	10.8
.223	21.0	.023	4.6	81.0	93.8	13.8
.221	22.0	.023	4.6	81.4	94.0	13.9
.223	20.0	.060	1.8	79.1	94.2	16.0
.220	23.0	.060	1.8	82.5	99.2	17.0
.222	21.0	.125	.8	79.4	103.0	23.5
.223	21.0	.125	.8	78.5	101.2	23.4
.268	31.0	$<.0005$	∞	70.6	74.9	5.6
.222	31.5	$<.0005$	∞	72.0	78.0	4.3
.220	33.0	.010	10.6	83.1	87.6	7.9
.223	30.0	.010	10.6	79.7	85.7	7.6
.222	31.0	.023	4.6	84.9	95.9	11.8
.220	32.0	.023	4.6	85.6	97.5	12.1

TABLE I

RESULTS OF NOTCHED-BAR TENSILE TESTS ON 24S-T ROD - Concluded

Notch diameter (in.)	Notch depth (percent)	Notch radius (in.)	Notch sharpness, a/R	Notch strength (psi)	Fracture stress (psi)	Notch ductility (percent)
0.198	49.0	<.0005	∞	79.6×10^3	85.2×10^3	7.07
.193	51.0	<.0005	∞	82.9	88.1	5.69
.222	51.5	<.0005	∞	78.7	85.5	8.1
.203	49.5	.002	53.0	83.6	90.3	7.39
.207	49.5	.002	53.0	84.4	91.9	8.42
.195	50.0	.003	35.4	89.4	98.5	9.84
.194	50.0	.003	35.4	88.7	96.0	8.25
.190	52.0	.007	15.2	92.1	100.9	8.72
.195	50.0	.007	15.2	86.7	93.9	8.11
.212	50.0	.007	15.2	88.0	96.5	9.43
.212	50.0	.010	10.6	87.3	98.2	10.8
.212	50.0	.010	10.6	87.0	99.3	12.3
.210	51.0	.010	10.6	90.0	97.3	7.5
.210	51.0	.010	10.6	90.8	99.7	9.05
.221	51.0	.010	10.6	89.1	95.5	7.1
.220	51.0	.010	10.6	90.0	98.2	8.1
.194	50.0	.017	6.2	92.2	101.7	9.47
.193	51.0	.017	6.2	92.4	102.7	9.78
.222	51.0	.023	4.6	92.8	104.2	11.1
.222	51.0	.023	4.6	93.8	105.4	11.2
.194	51.0	.029	3.7	93.4	106.0	12.3
.192	51.0	.029	3.7	94.5	108.0	12.7
.211	51.0	.045	2.4	93.0	112.2	17.4
.212	50.0	.045	2.4	92.3	112.0	17.9
.213	49.5	.060	1.8	91.1	113.0	19.8
.213	49.5	.060	1.8	91.0	111.5	18.6
.213	49.5	.125	.8	86.5	109.2	21.8
.214	49.5	.125	.8	85.5	113.0	25.8
.212	50.0	.250	.4	81.0	107.0	25.5
.213	49.5	.250	.4	80.6	104.5	24.0
.211	50.0	2.000	.05	72.3	102.1	32.6
.211	50.0	2.000	.05	71.4	101.4	33.1
.223	60.5	<.0005	∞	83.3	87.4	4.7
.223	60.5	<.0005	∞	83.1	85.8	3.5
.221	61.2	.010	10.6	94.7	101.2	6.6
.222	60.9	.010	10.6	95.5	103.5	7.8
.222	60.8	.023	4.6	98.3	110.2	10.6
.223	60.6	.023	4.6	98.0	109.4	10.4
.221	80.0	<.0005	∞	92.3	95.4	3.7
.221	81.0	<.0005	∞	93.5	98.0	4.7
.225	80.0	.005	21.2	100.0	105.2	4.7
.224	80.0	.005	21.2	98.0	102.2	4.38
.222	80.0	.010	10.6	104.3	114.3	8.6
.217	81.0	.010	10.6	102.4	110.3	7.0
.223	80.0	.023	4.6	106.0	119.6	11.2
.223	80.0	.023	4.6	106.3	120.9	12.0
.226	80.0	.045	2.4	99.4	120.2	18.2
.226	80.0	.045	2.4	99.3	120.0	17.7
.225	80.0	.060	1.8	94.0	115.3	19.0
.225	80.0	.060	1.8	94.0	116.5	20.2
.221	80.0	.060	1.8	96.5	119.8	19.9
.226	80.0	.060	1.8	91.5	112.5	18.6
.223	80.0	.125	.8	88.8	111.5	20.7
.224	80.0	.125	.8	87.5	110.8	21.4
.224	80.0	.125	.8	84.0	106.8	22.1
.224	80.0	.125	.8	85.0	108.9	22.9
.224	80.0	.250	.4	81.2	106.0	25.0
.224	80.0	.250	.4	80.2	104.2	23.6

TABLE II

RESULTS OF NOTCHED-BAR TENSILE TESTS ON 24S-T ROD

$\frac{3}{4}$ -inch rod re-solution-heat-treated before
machining; $a = 0.106$ inch]

Notch diameter (in.)	Notch depth (percent)	Notch radius (in.)	Notch sharpness, a/R	Notch strength (psi)	Fracture stress (psi)	Notch ductility (percent)
0.209	0	∞	0	71.2×10^3	102.8×10^3	34.9
.215	49.0	<.0005	∞	78.6	83.2	5.49
.213	50.0	<.0005	∞	79.6	83.6	5.00
.207	49.0	.002	53.0	79.5	83.5	4.95
.207	50.0	.002	53.0	81.4	87.2	6.95
.211	49.0	.002	53.0	82.5	89.1	7.8
.210	51.0	.002	53.0	82.7	90.2	8.31
.211	51.0	.007	15.2	84.8	93.7	9.2
.209	51.0	.007	15.2	87.2	93.8	7.47
.213	49.0	.010	10.6	88.4	95.0	7.42
.213	50.0	.010	10.6	86.9	93.0	6.95
.213	50.0	.017	6.2	90.6	99.5	8.65
.212	50.0	.017	6.2	91.5	102.8	11.15
.214	49.0	.017	6.2	89.1	101.8	12.5
.211	50.0	.023	4.6	92.2	103.0	10.1
.212	49.0	.023	4.6	93.0	104.5	11.05
.212	49.0	.029	3.7	94.5	107.5	12.2
.214	49.0	.029	3.7	92.0	105.5	12.9
.213	50.0	.045	2.4	90.2	108.2	16.9
.196	50.0	.045	2.4	89.0	108.8	18.9
.210	51.0	.045	2.4	92.3	108.9	15.7
.213	49.0	.060	1.8	90.8	110.2	17.8
.213	49.0	.060	1.8	91.4	109.5	16.6
.196	49.0	.125	.8	84.2	108.9	23.8
.196	49.0	.125	.8	84.8	108.3	23.3
.212	50.0	2.000	.05	73.3	101.5	30.0
.211	50.0	2.000	.05	75.0	102.5	31.1

TABLE III

RESULTS OF ANALYSIS OF NECKING IN 24S-T ROD BY
USING INSTABILITY METHOD

$\left[\frac{3}{4} \right]$ - inch rod re-solution-heat-treated after
machining; $a = 0.106$ inch

Notch radius (in.)	Notch depth (percent)	Necking strain (percent)
∞	0	18.6
.250	50.0	18.2
.250	80.0	18.2
.125	20.0	18.5
.125	50.0	18.3
.125	80.0	18.5
.060	50.0	18.2



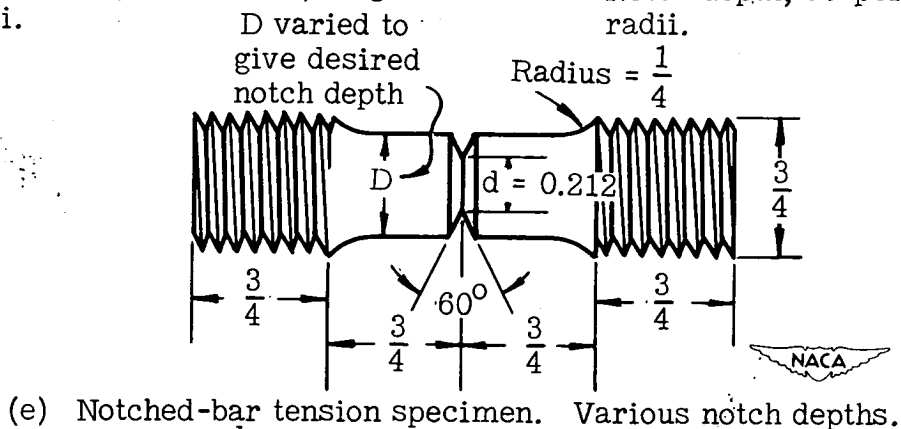
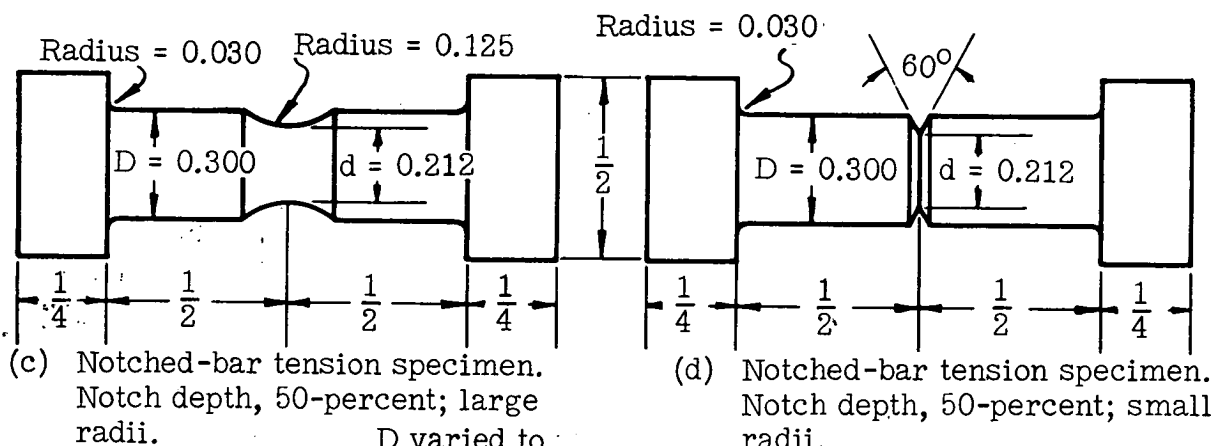
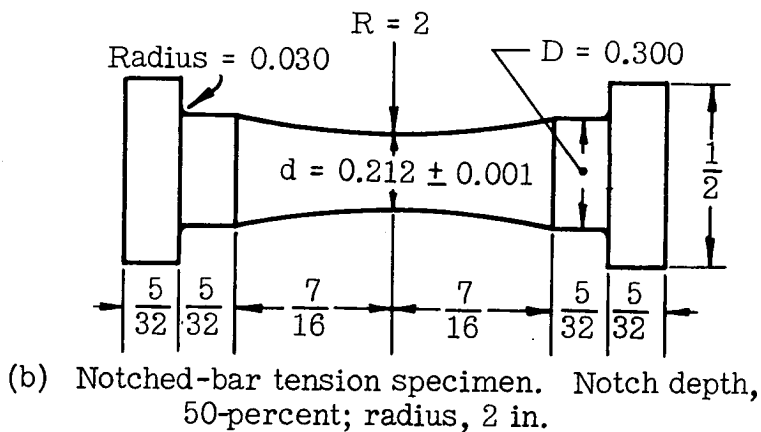
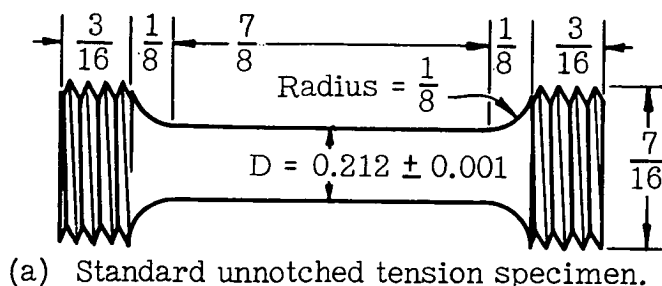


Figure 1.- Test specimens. All dimensions are in inches.

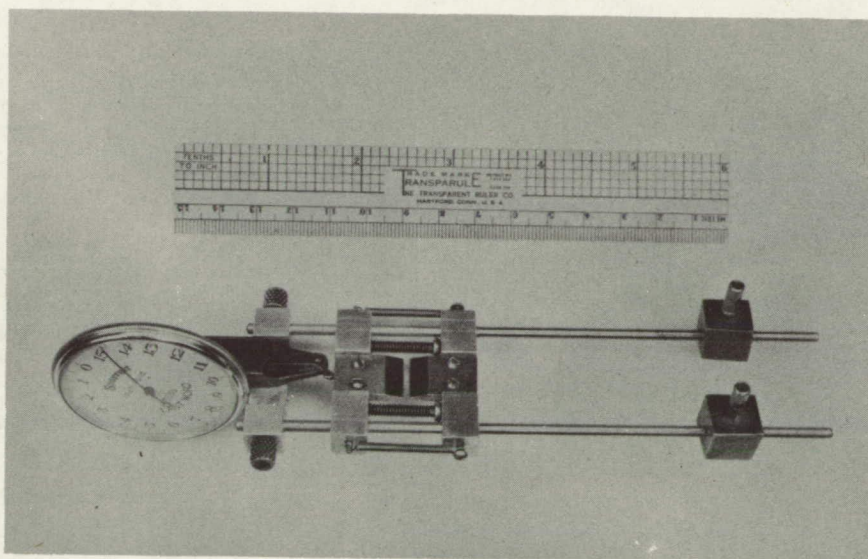
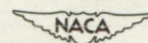


Figure 2.- Gage for measuring average transverse strain at root of notched section.



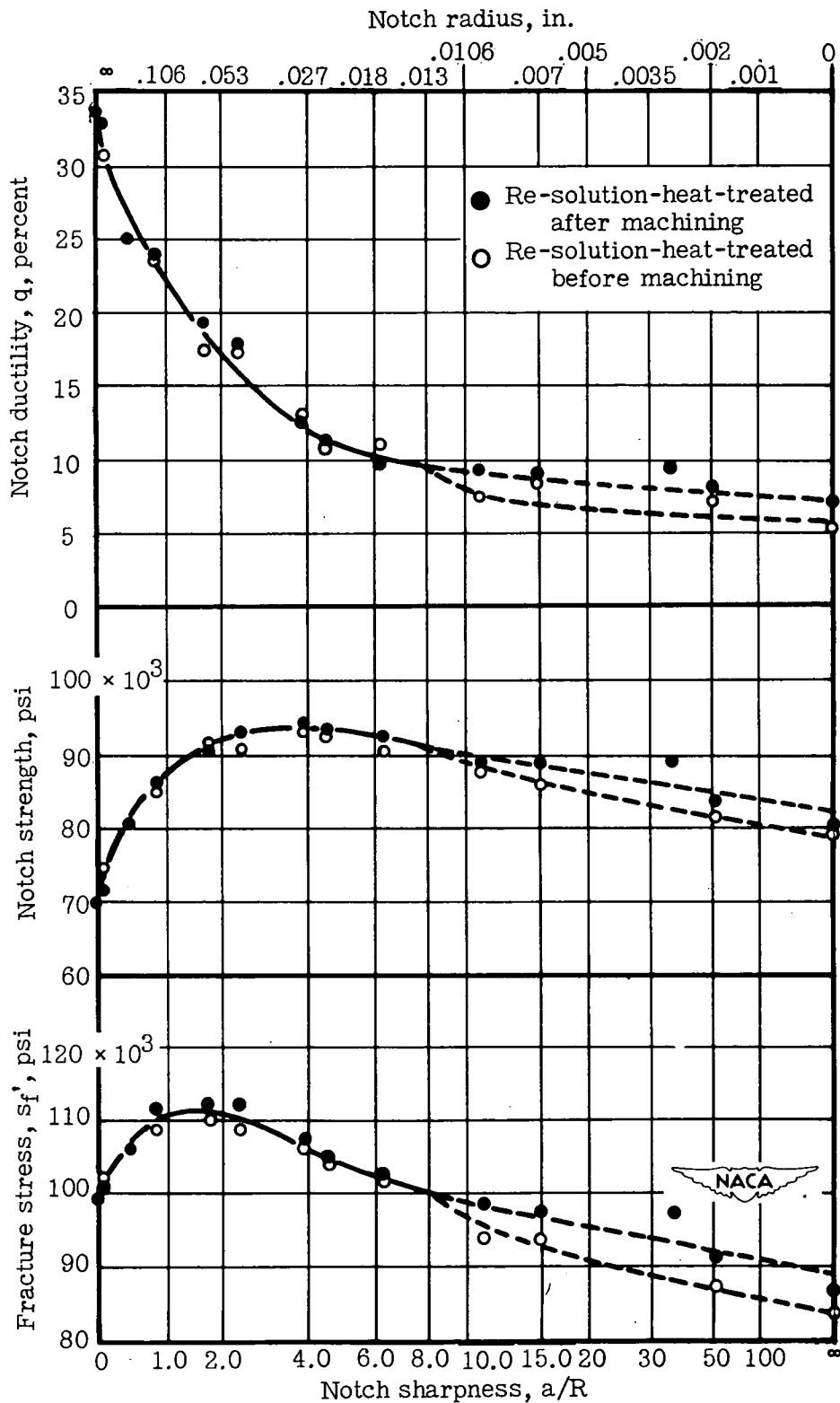


Figure 3.- Effect of notch sharpness on fracture stress, notch strength, and notch ductility of re-solution-heat-treated 24S-T aluminum-alloy rod. 50-percent, 60° , V-type notches. Each point is average of two or more tests. $a = 0.106$ inch.

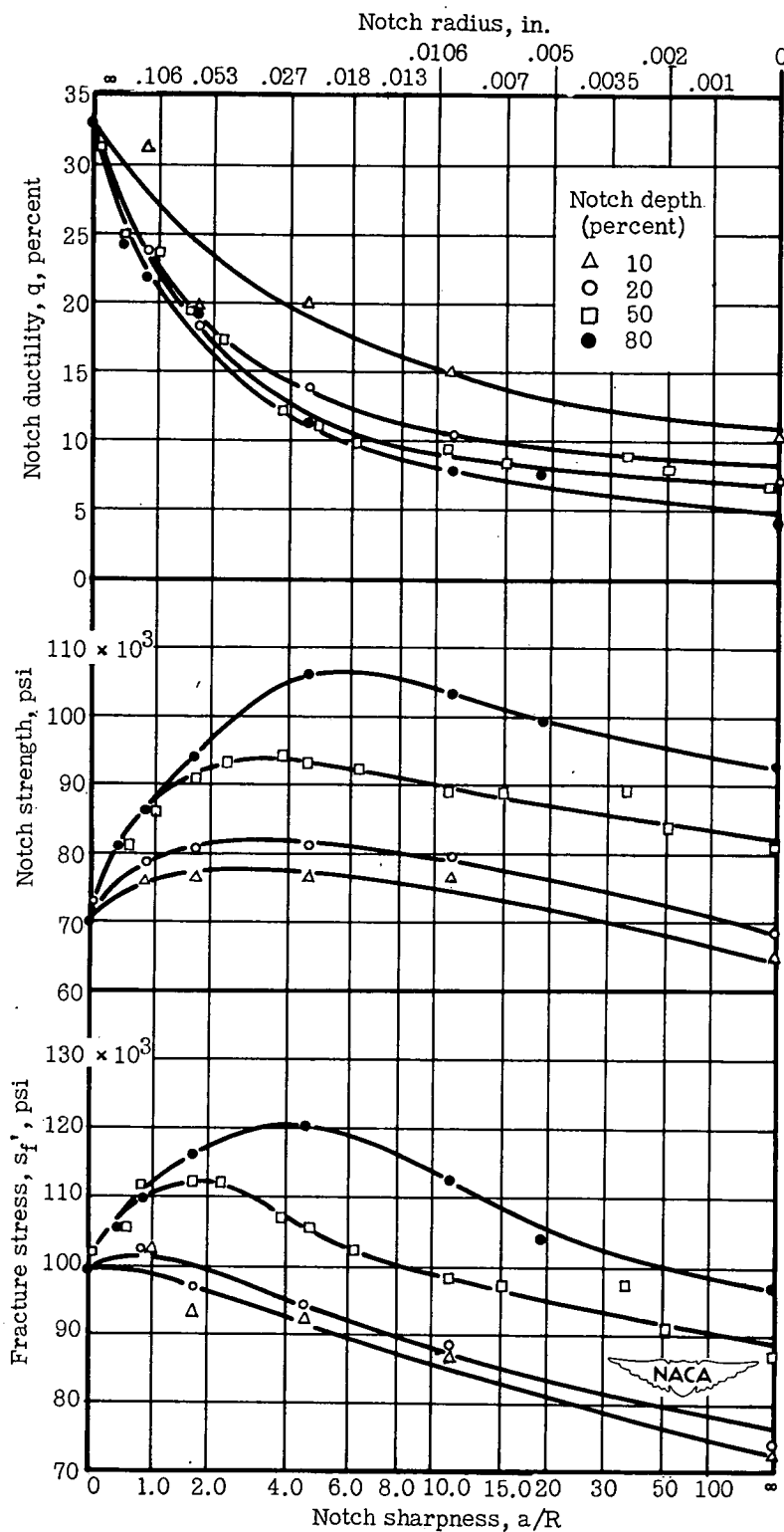


Figure 4.- Effect of notch sharpness on fracture stress, notch strength, and notch ductility of 24S-T aluminum-alloy rod re-solution-heat-treated after machining. 60° , V-type notches; various notch depths. Each point is average of two or more tests. $a = 0.106$ inch.

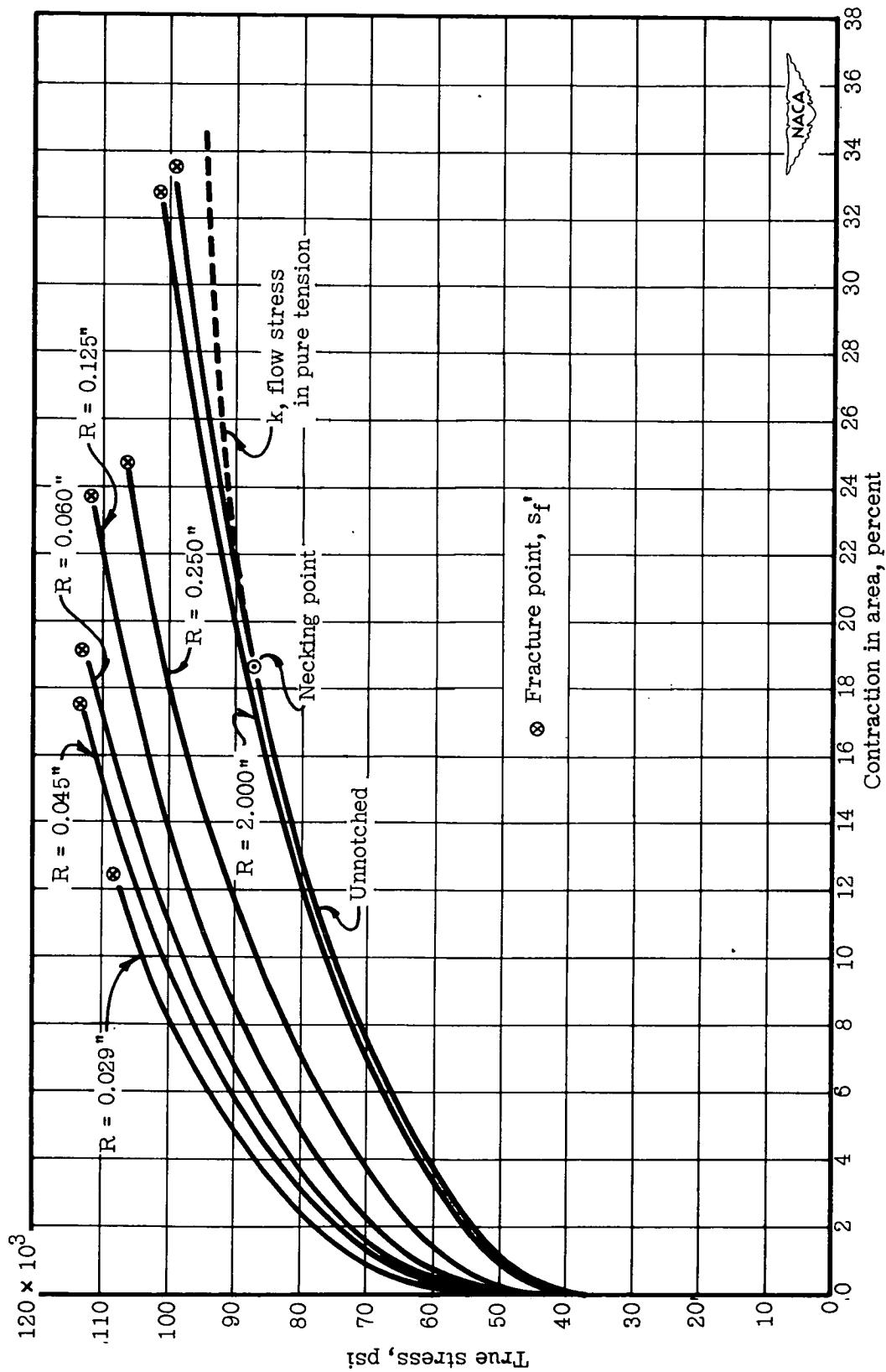


Figure 5.- Stress-strain curves for 24S-T aluminum-alloy rod re-solution-heat-treated after machining. 50-percent, 60°, V-type notches and various notch radii. $a = 0.106$ inch.

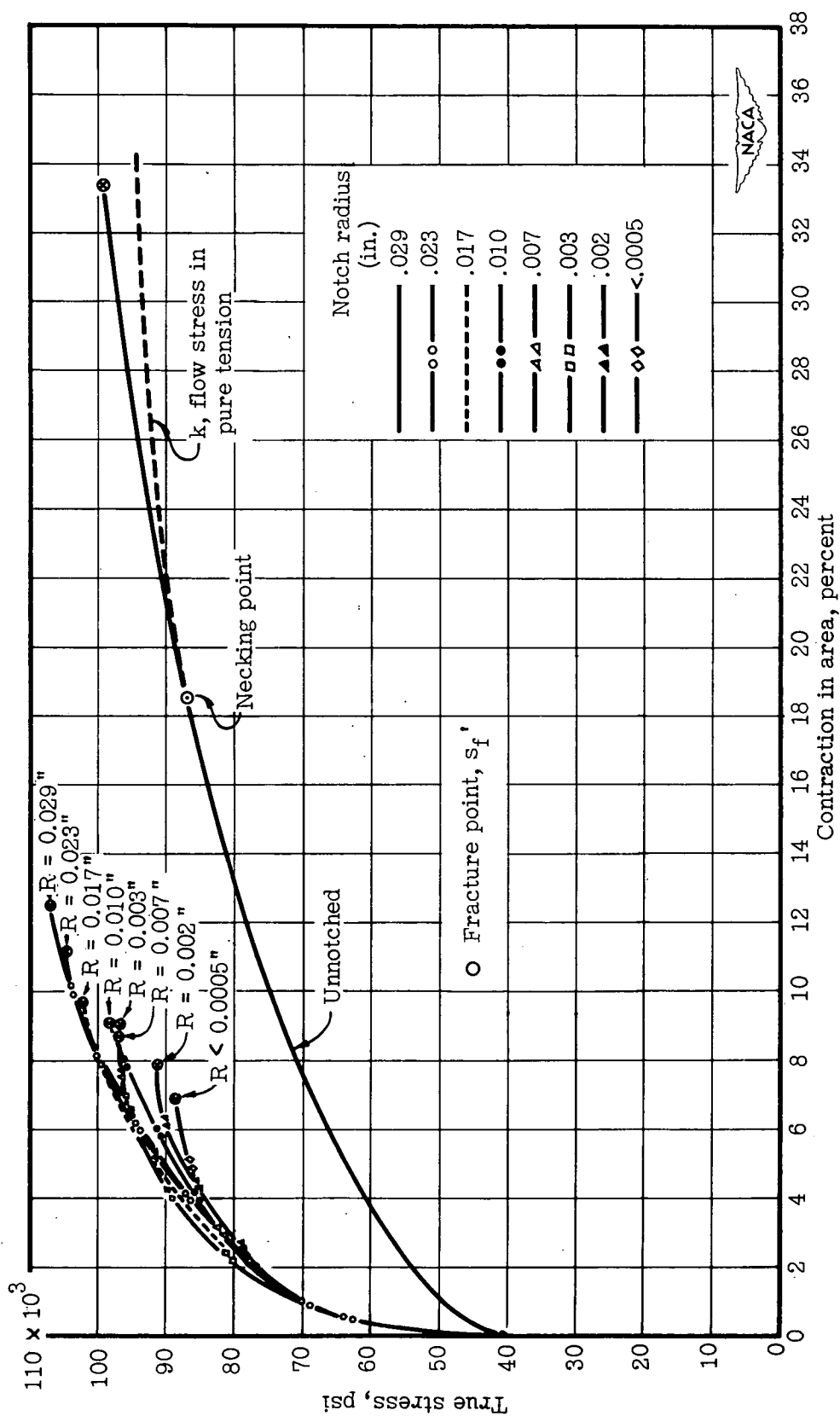


Figure 6.- Stress-strain curves for 24S-T aluminum-alloy rod re-solution-heat-treated after machining. 50-percent, 60°, V-type notches and various sharp notch radii. $a = 0.106$ inch.

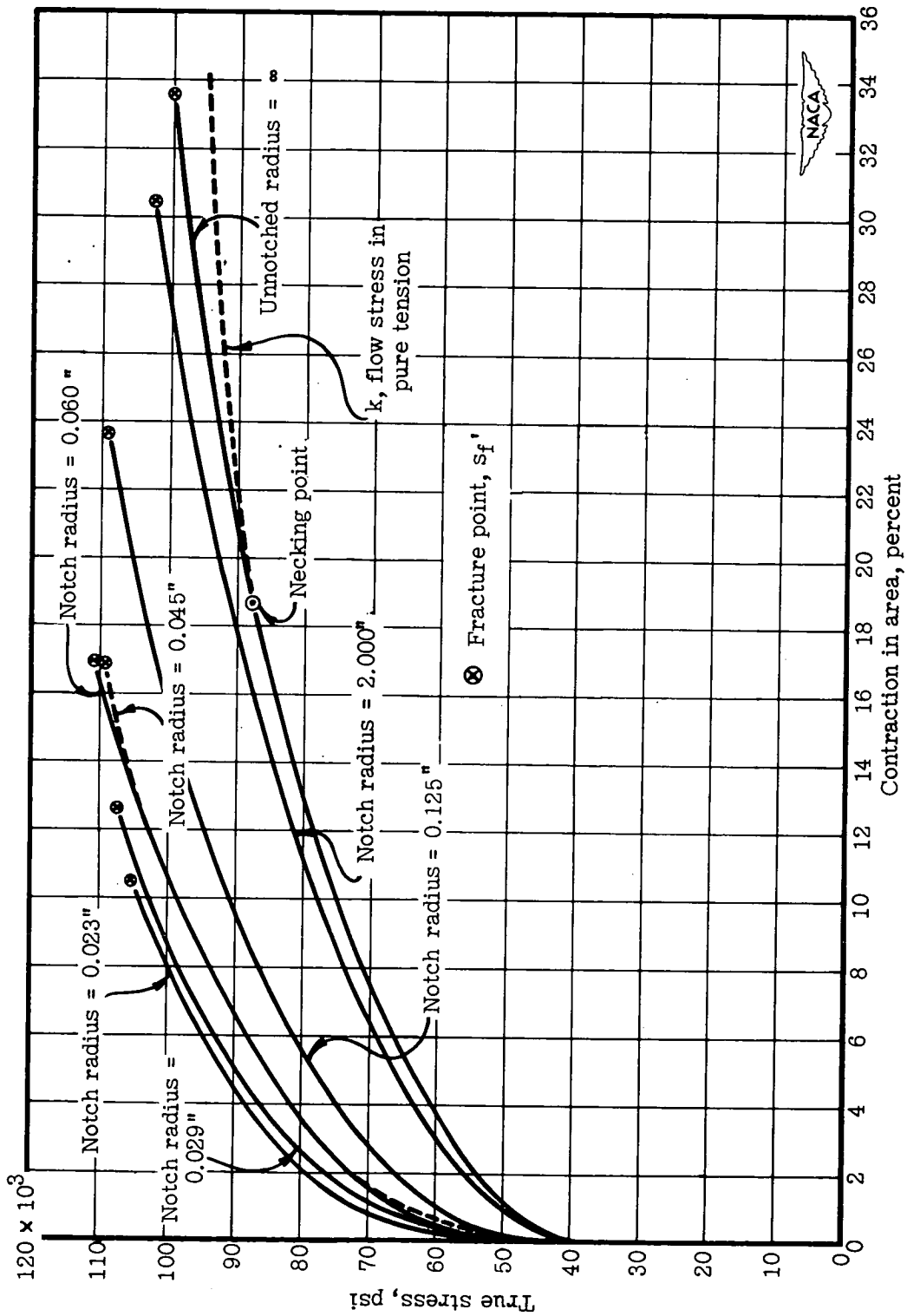


Figure 7.- Stress-strain curves for 24S-T aluminum-alloy rod re-solution-heat-treated before machining. 50-percent, 60°, V-type notches and various notch radii. $a = 0.106$ inch.

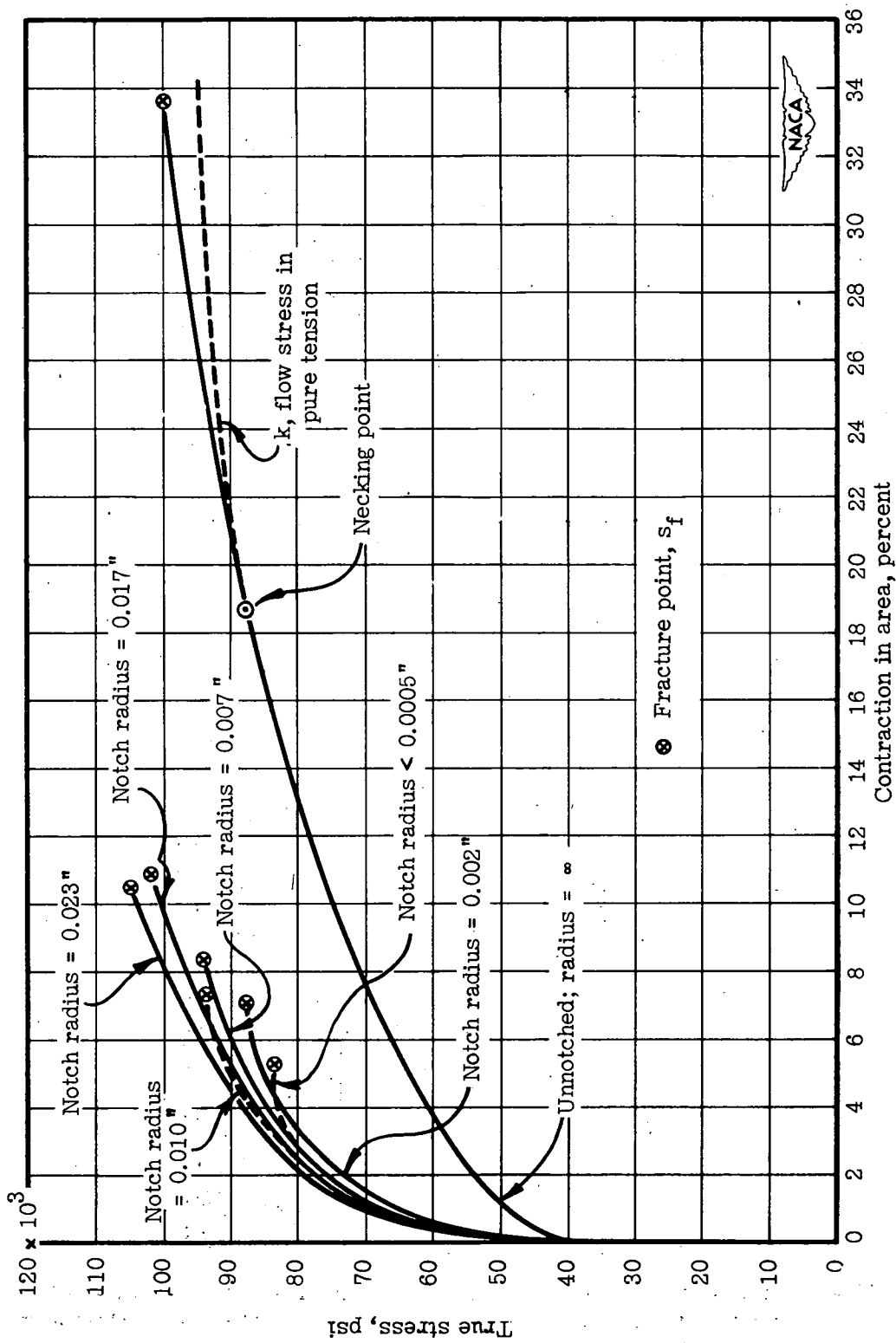


Figure 8.- Stress-strain curves for 24S-T aluminum-alloy rod re-solution-heat-treated before machining. 50-percent, 60°, V-type notches and various sharp notch radii. $a = 0.106$ inch.

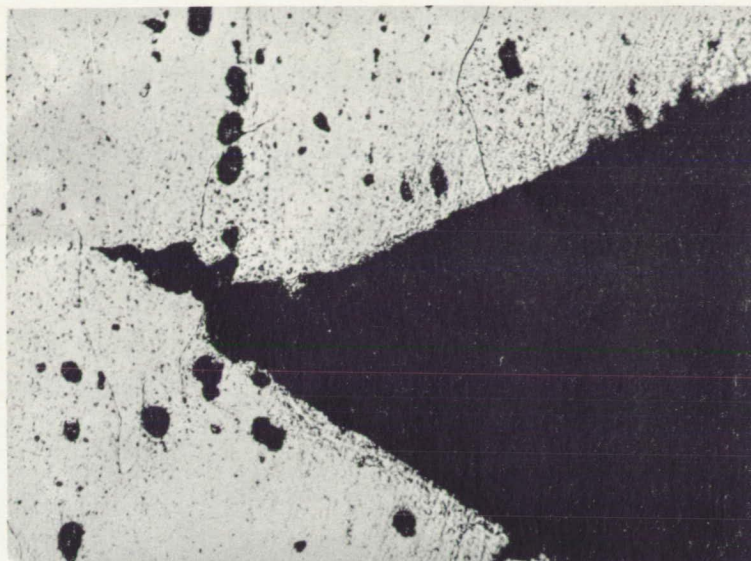


Figure 9.- Crack at notch bottom in 24S-T specimen with sharp notch radius (less than 0.0005-inch) and 50-percent notch depth. Specimen re-solution-heat-treated after machining. Magnified 500X.

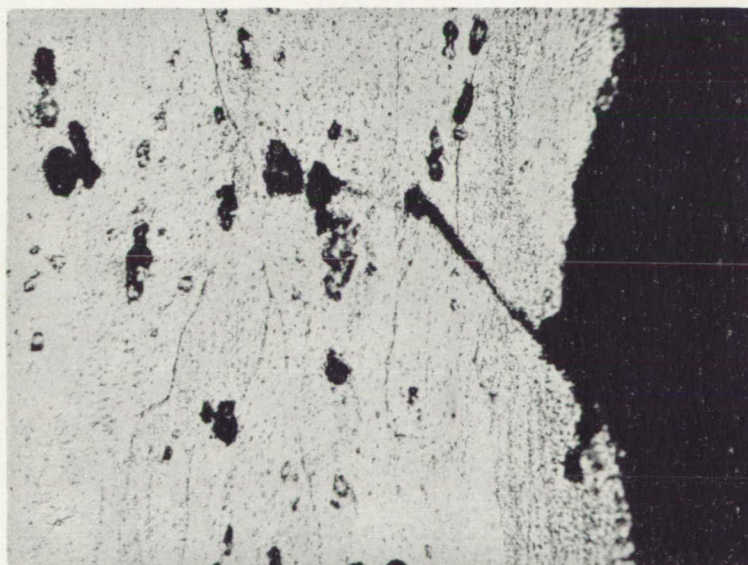
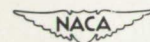
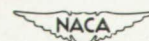


Figure 10.- Crack at notch bottom in 24S-T specimen with notch radius of 0.007 inch and 50-percent notch depth. Specimen re-solution-heat-treated after machining. Magnified 500X.



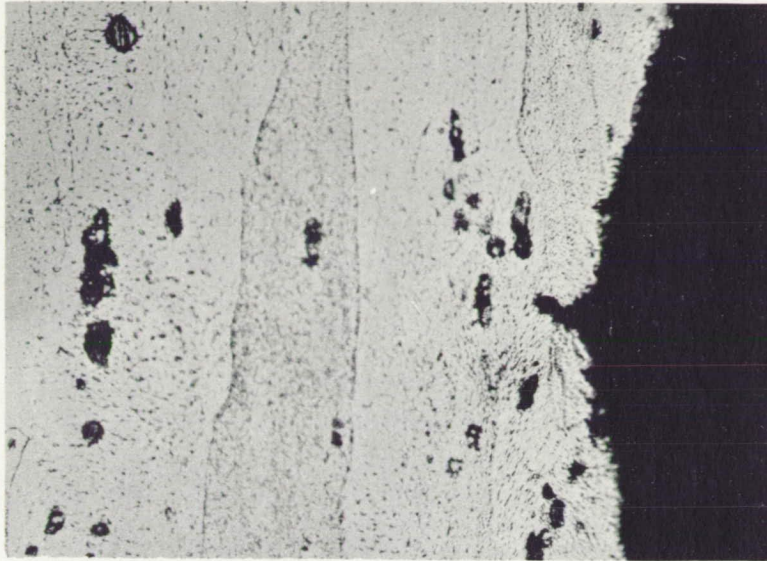
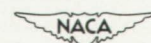


Figure 11.- Crack at notch bottom in 24S-T specimen with notch radius of 0.029 inch and 50-percent notch depth. Specimen re-solution-heat-treated after machining. Magnified 500X.



Figure 12.- Notch bottom of 24S-T specimen with notch radius of 0.125 inch and 50-percent notch depth showing absence of crack. Specimen re-solution-heat-treated after machining. Magnified 100X.



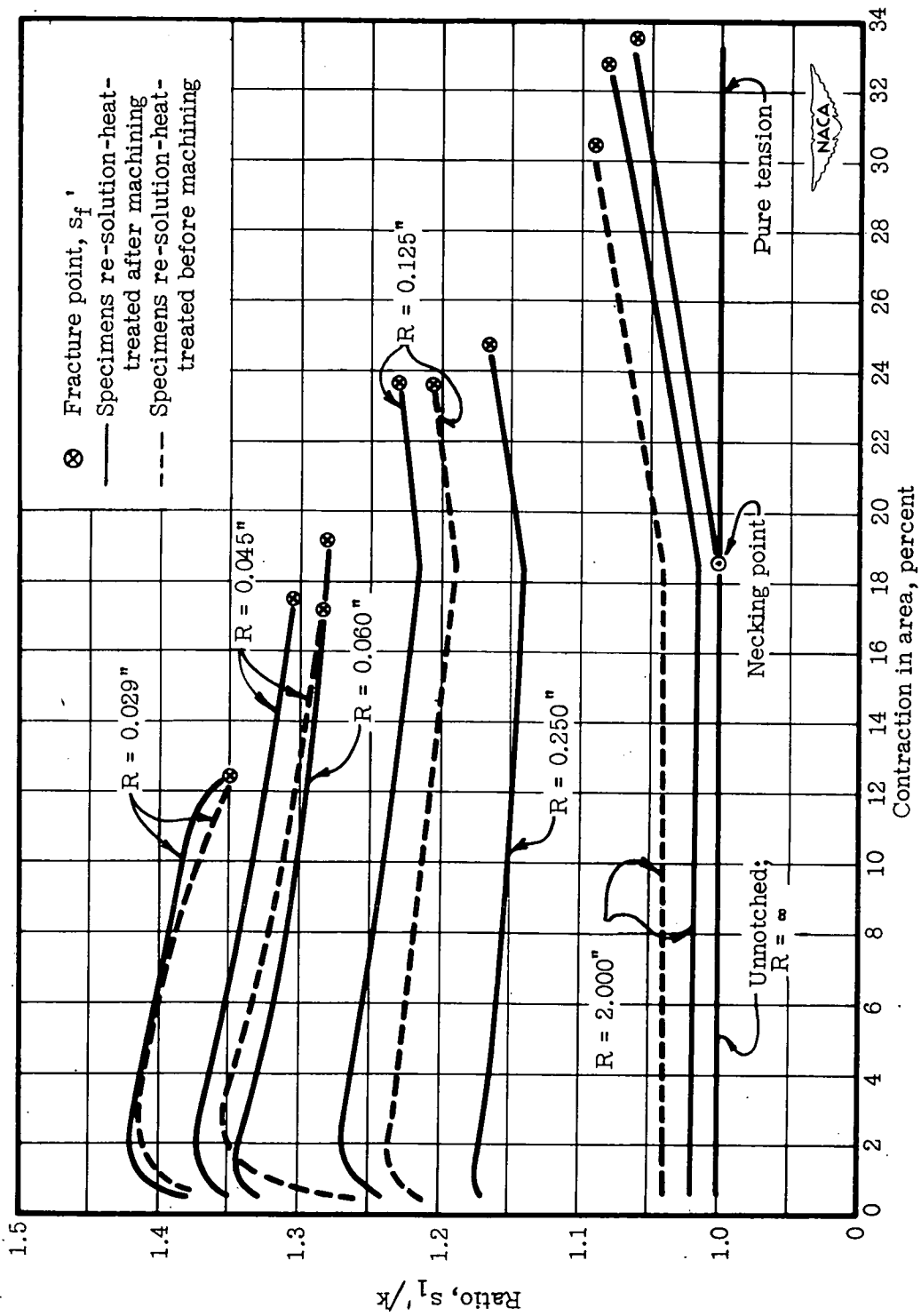


Figure 13.- Effect of progressive strain on ratio of average longitudinal stress to flow stress for 24S-T aluminum-alloy rod. 50-percent, 60°, V-type notches and various notch radii. $a = 0.106$ inch.

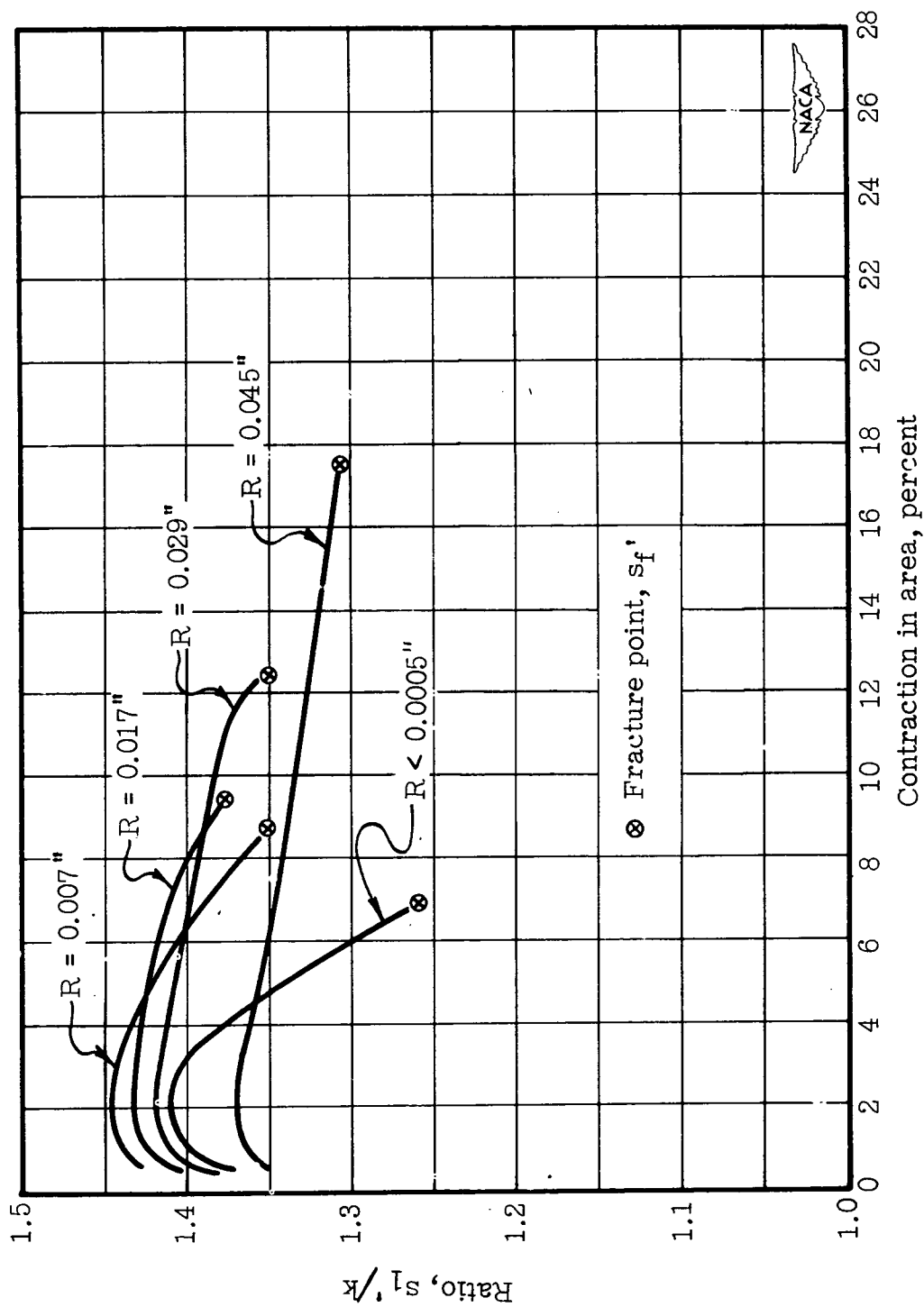


Figure 14.- Effect of progressive strain on ratio of average longitudinal stress to flow stress for 24S-T aluminum-alloy rod re-solution-heat-treated after machining. 50-percent, 60°, V-type notches and various sharp notch radii. $a = 0.106$ inch.

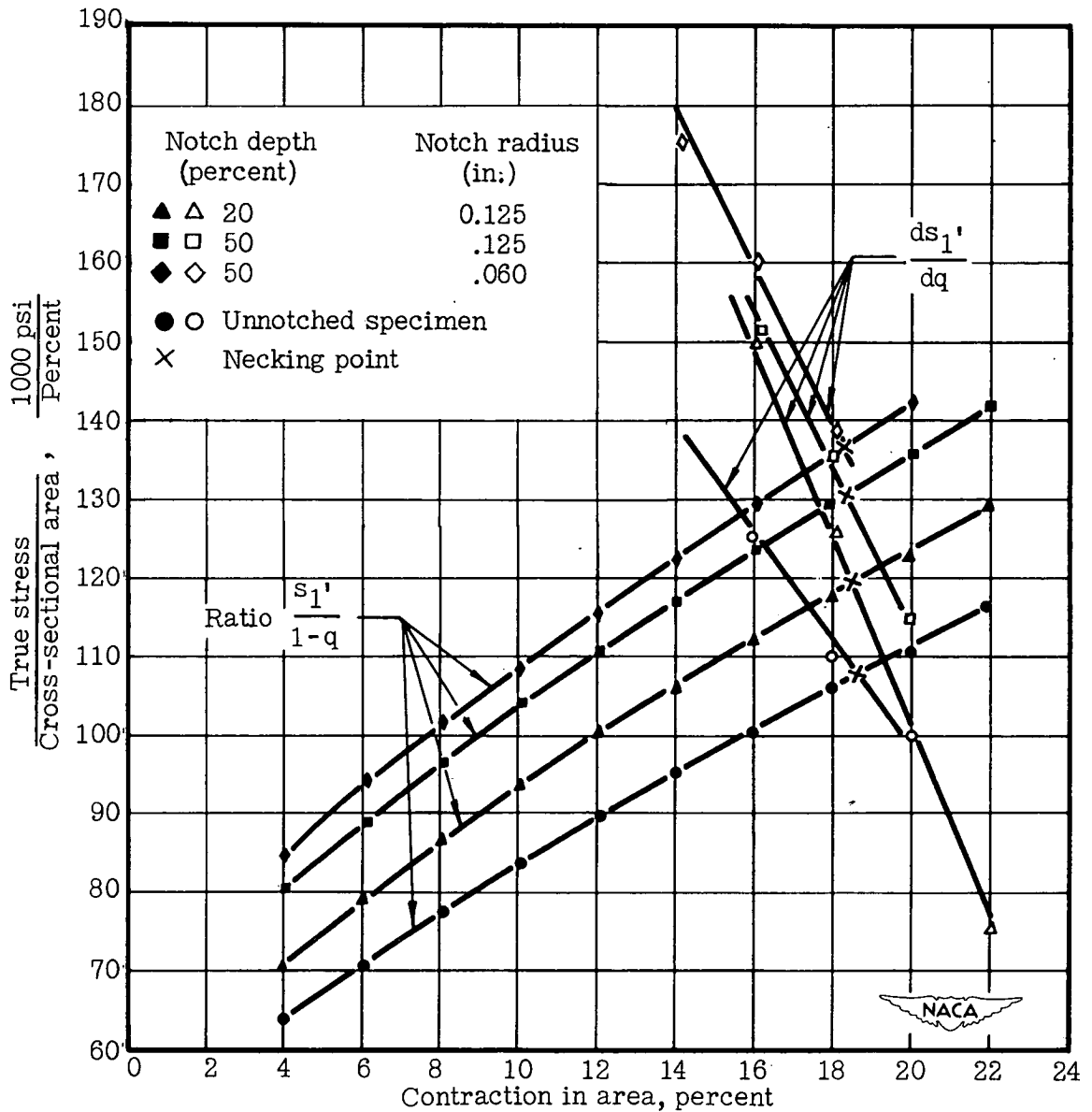


Figure 15.- Condition for necking of $\frac{3}{4}$ -inch 24S-T aluminum-alloy rod re-solution-heat-treated after machining. Various notch depths and notch radii. Equality of $\frac{ds_1'}{dq}$ and $\frac{s_1'}{1-q}$ is shown. s_1' , true stress.

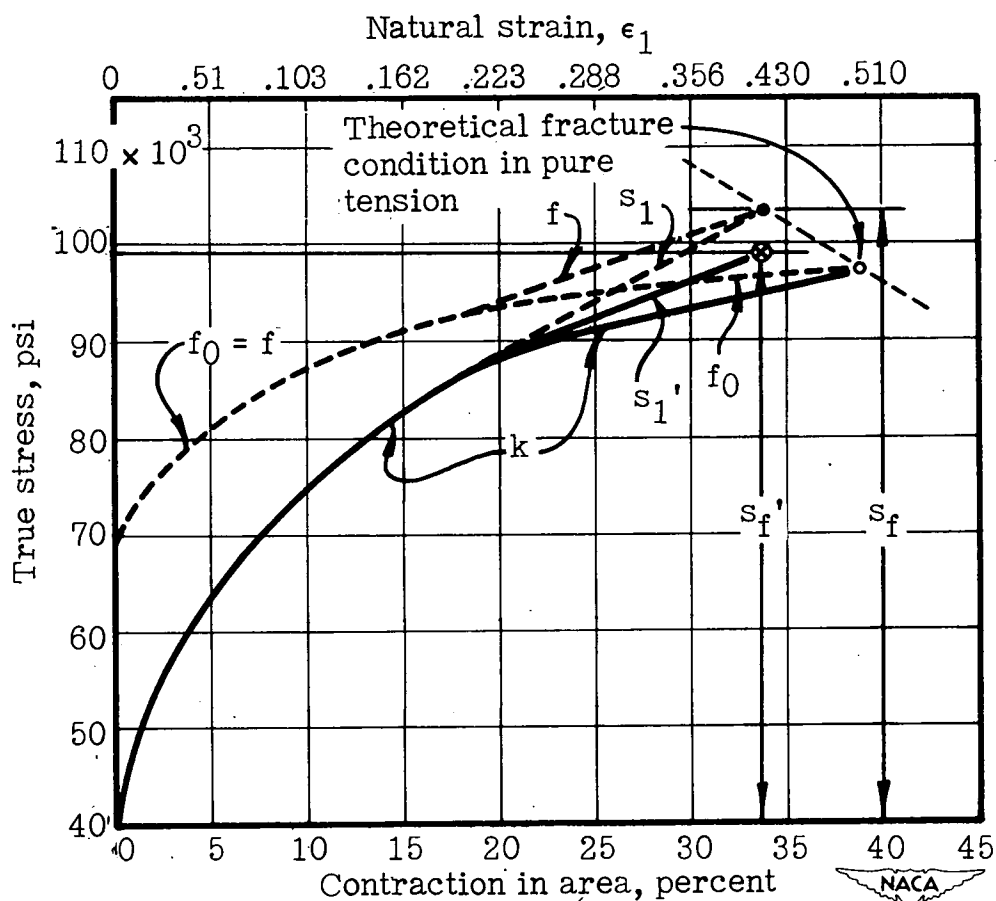


Figure 16.- Relation between stress required for plastic flow s and fracture stress f for various strains. Derived from tensile test data. 24S-T rod re-solution-heat-treated.

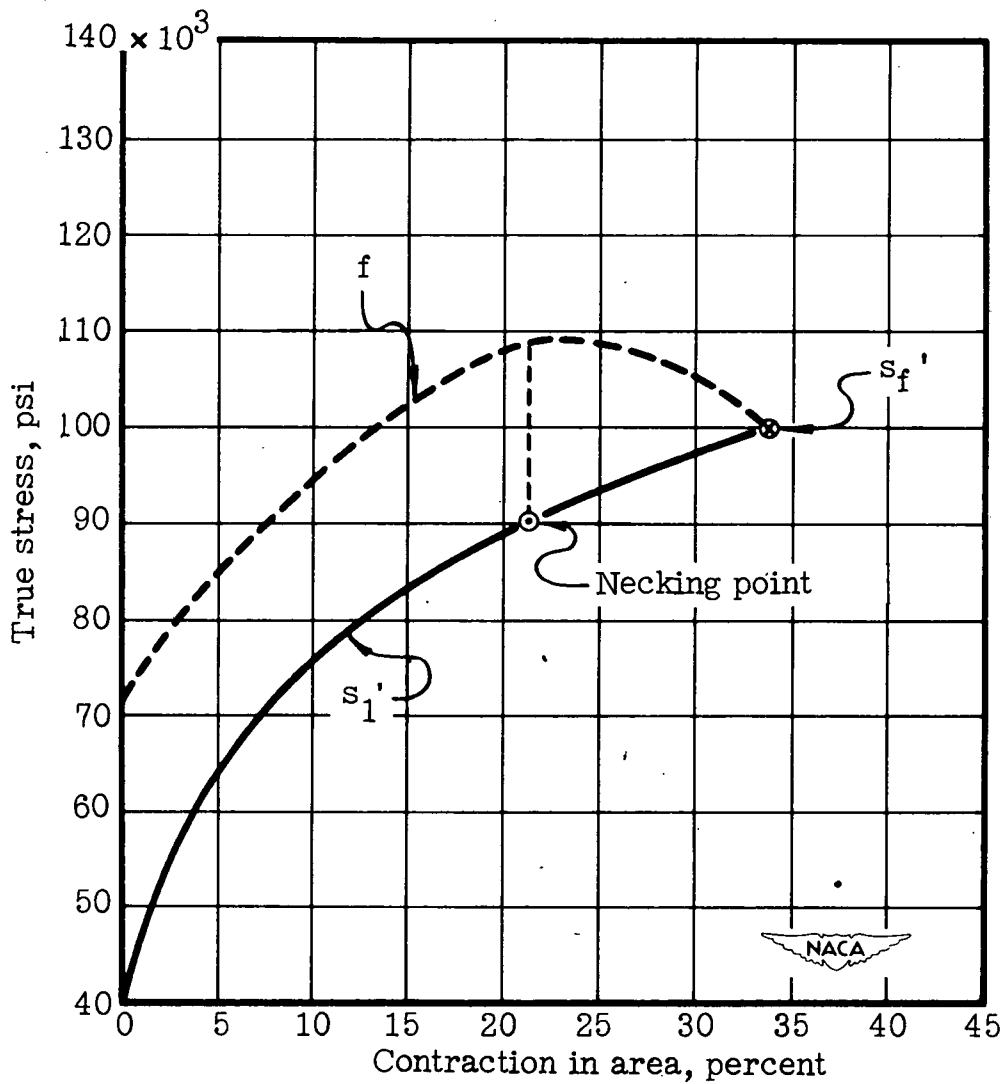


Figure 17.- Schematic representation of Kuntze's method for showing relation between ratio of stress required for plastic flow s_1' and fracture stress f for various strains.

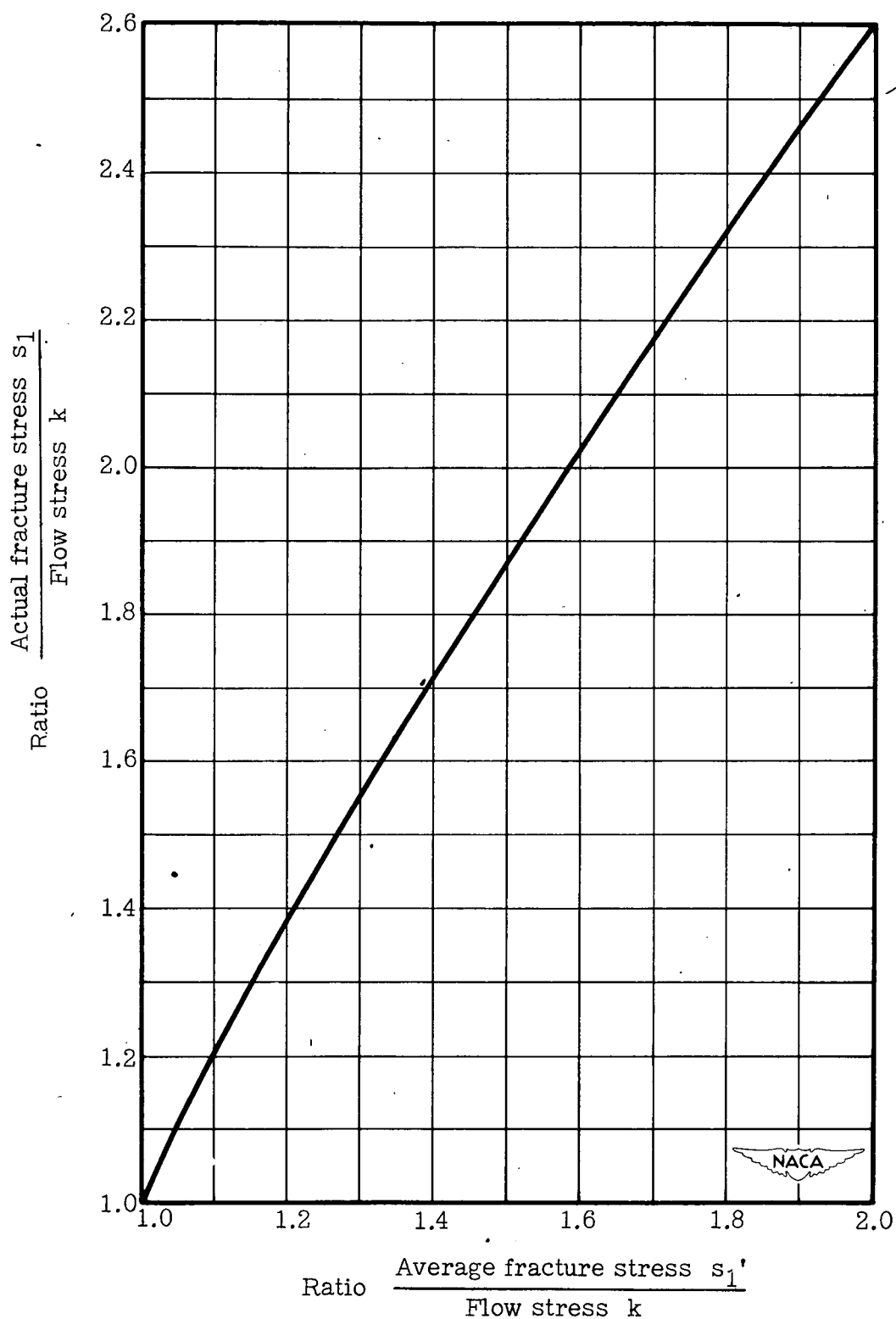


Figure 18.- Relation between average triaxiality s_1'/k and actual triaxiality s_1/k from Bridgman's equations for a necked tensile specimen.

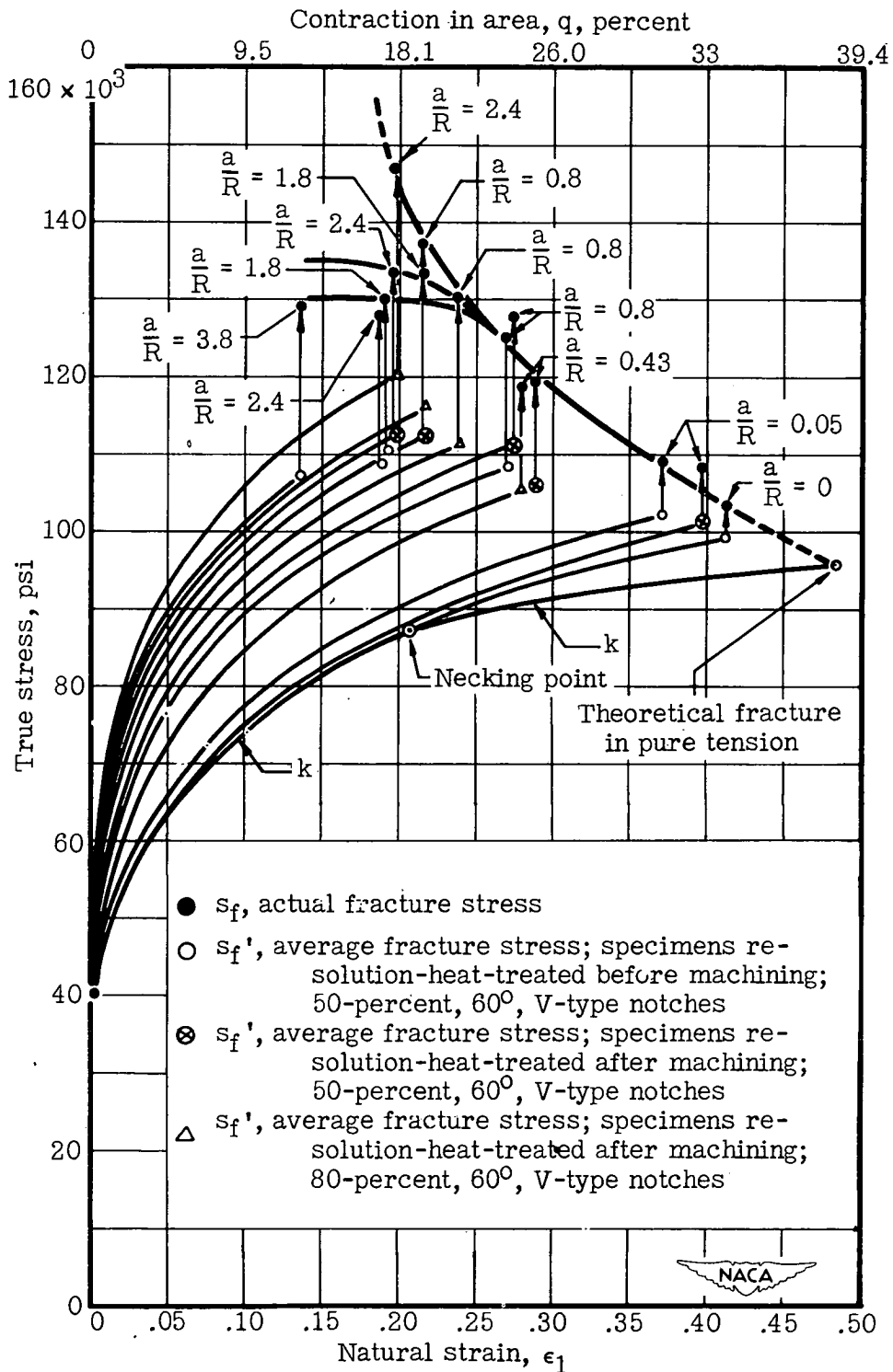


Figure 19.- Average stress-strain curves for various notch depths and radii with actual fracture stresses plotted vertically above average fracture points. 24S-T rod; $a = 0.106$ inch.

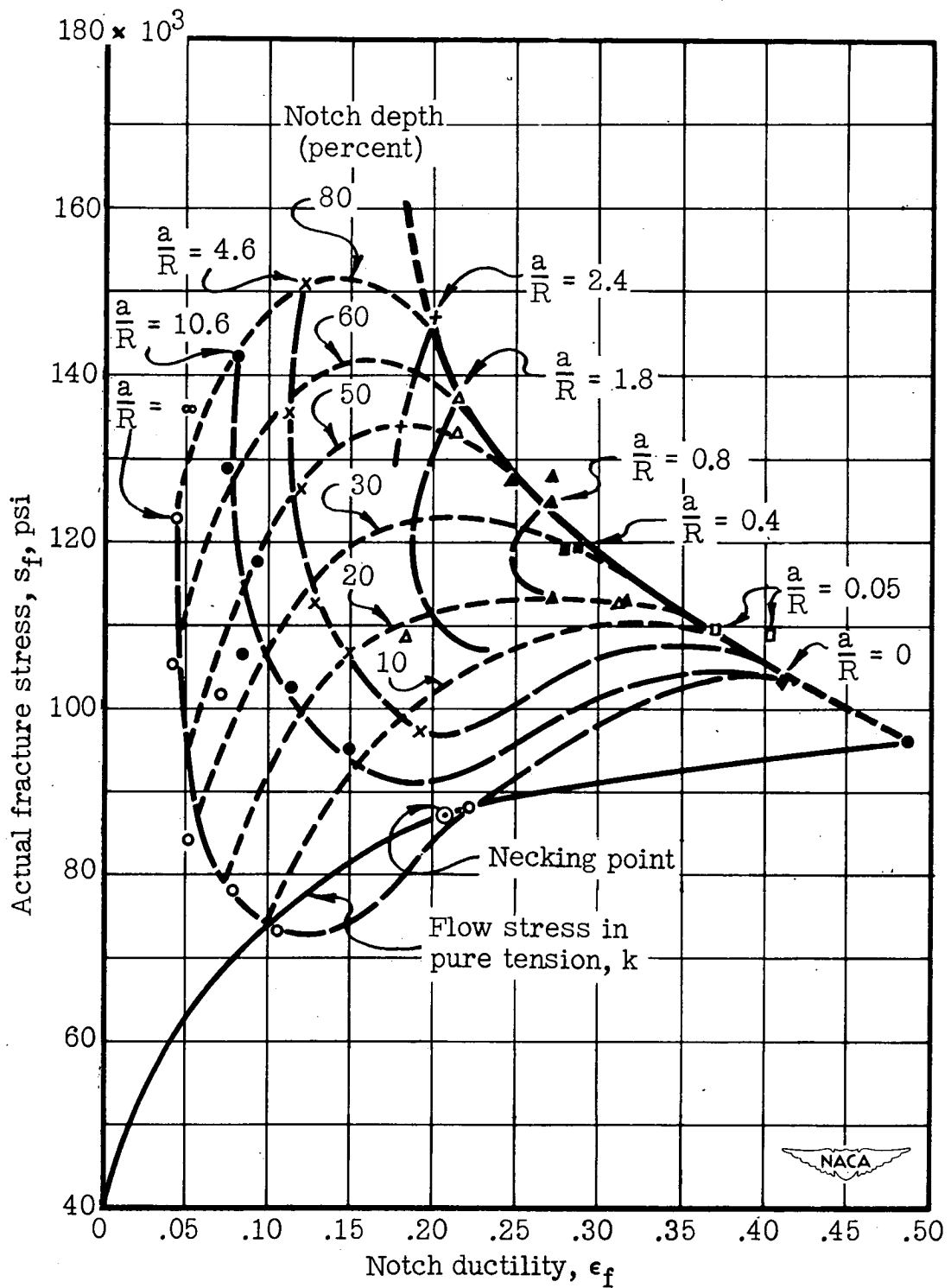


Figure 20.- Relation between actual fracture stress s_f and fracture strain ϵ_f for all notch depths and notch radii investigated. Re-solution-heat-treated 24S-T rod; $a = 0.106$ inch.

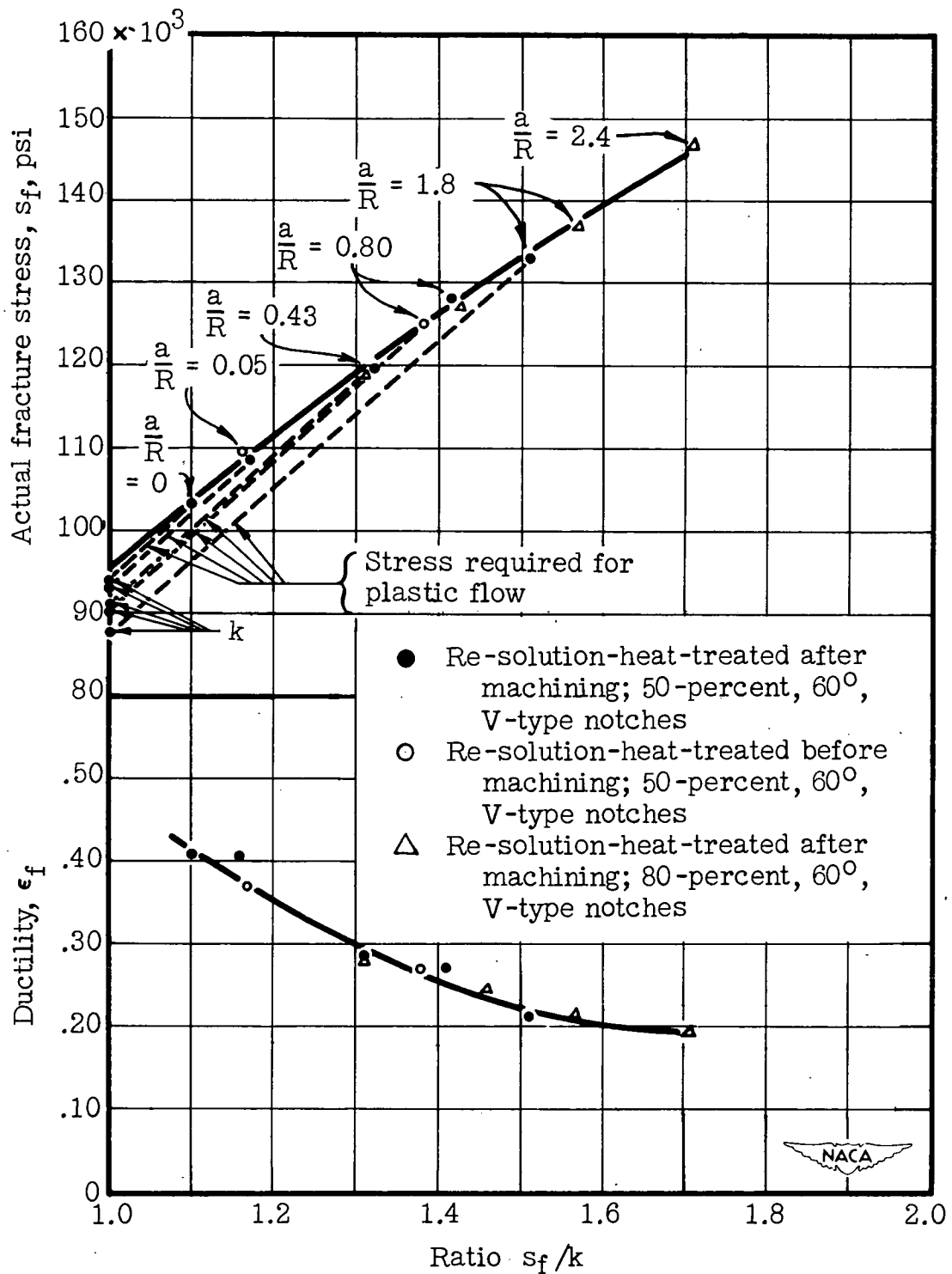


Figure 21.- Effect of degree of triaxiality s_f/k on actual fracture stress s_f , stress required for plastic flow, and fracture ductility ϵ_f . 24S-T rod re-solution-heat-treated. $a = 0.106$ inch.

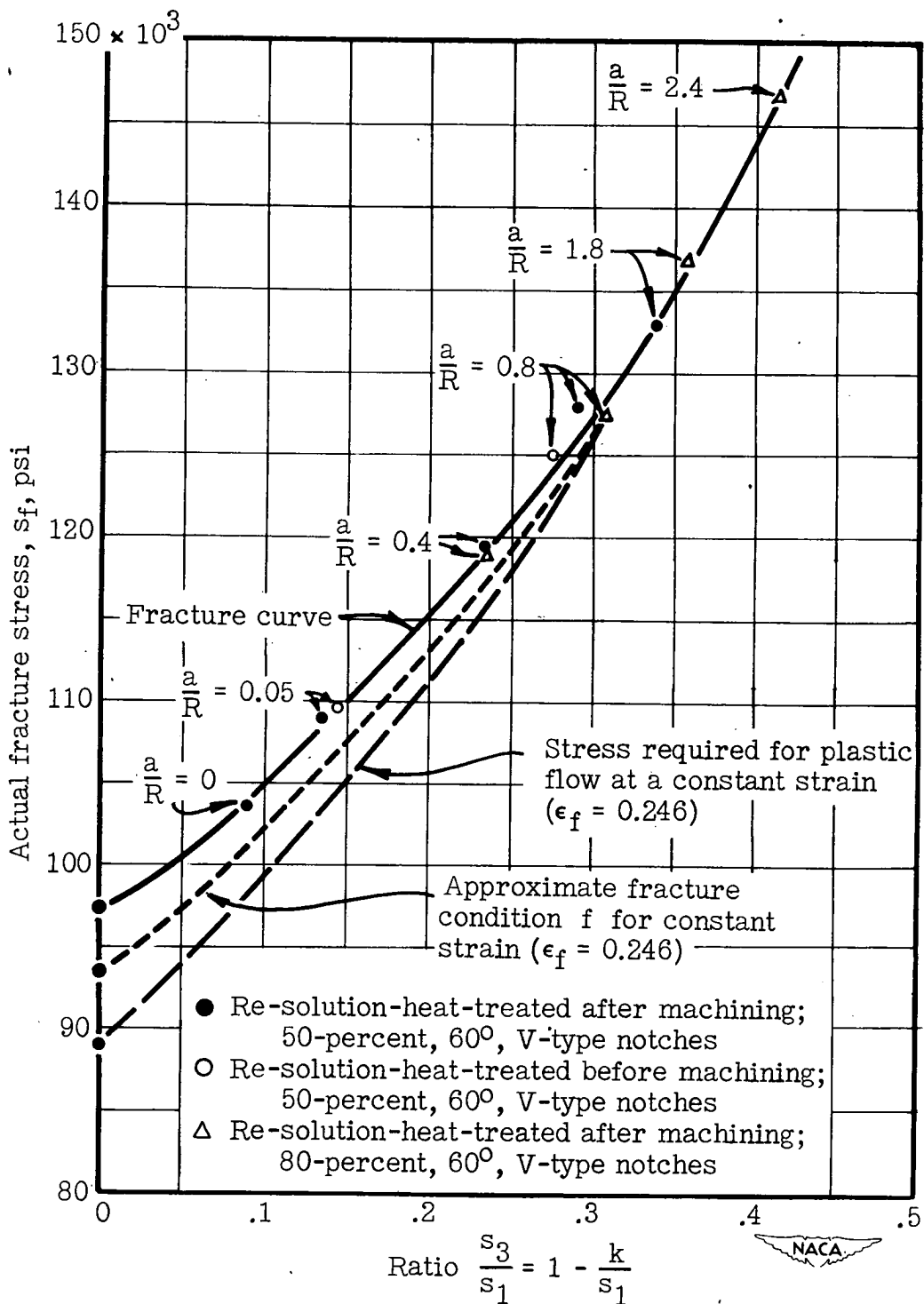


Figure 22.- Effect of degree of triaxiality s_3/s_1 on actual fracture stress s_f , stress required for plastic flow at a constant strain, and approximate fracture condition f at a constant strain. 24S-T rod re-solution-heat-treated. $a = 0.106$ inch.

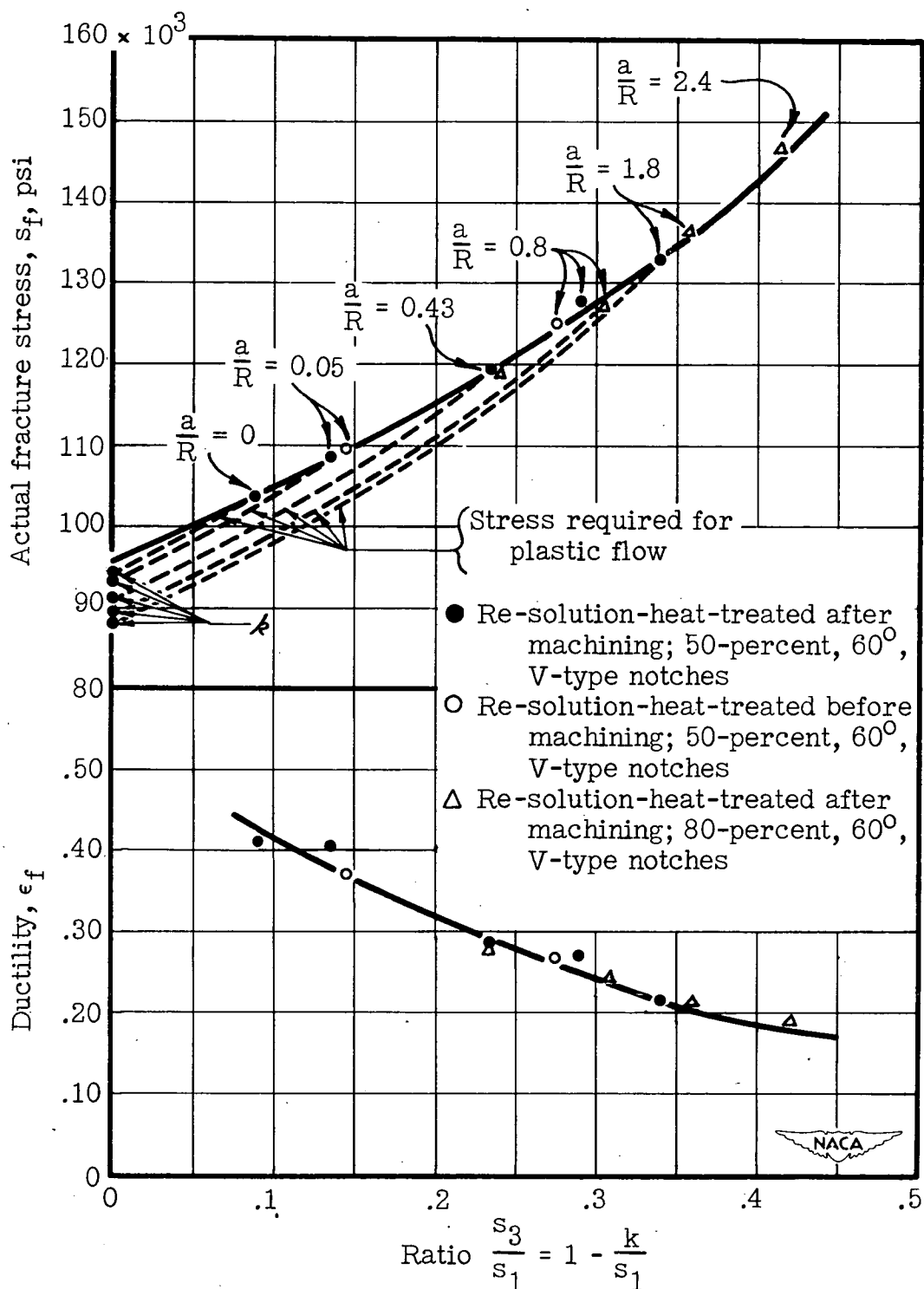


Figure 23.- Effect of degree of triaxiality on actual fracture stress s_f , stress required for plastic flow, and fracture ductility ϵ_f . 24S-T rod.

$a = 0.106$ inch.